

# **A Practical Guide to Multi-Criteria Decision Analysis**

**A Workbook Companion to V·I·S·A**

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## Preface

This workbook has been set up to provide you with the fundamental concepts of the MCDA approach and the V·I·S·A software package.

The structure of this workbook is to provide you with a brief introduction to the theory, followed by a few focusing questions that will help guide your thinking. Several small examples will be used to illustrate the concepts, and a medium-sized example is presented for you both during and at the end of this Workbook. This latter example will take you through all the V·I·S·A model creation and analysis steps in turn. The aim of the workbook is to provide you with the capability and means to create your own V·I·S·A model, and therefore your own assessment of a decision problem.

It is not necessary to read or work through all the examples and questions. If you feel comfortable with the concepts being explored then please skip those parts. The most important aspects and fundamental ideas are highlighted by the symbol  $\mathbb{K}$ . Please read these parts, as they will help you to create your own fully operational model.

$\mathbb{K}$  In the back of this workbook you will find a *Quick Guide to V·I·S·A* which briefly gives the instructions for putting together a basic model using the V·I·S·A package.

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## Introduction to V·I·S·A

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### *Objectives:*

- Students will gain an understanding of the MCDA approach and its process.
  - Students will gain an understanding of the features of the V·I·S·A software package and its link to the MCDA approach.
  - Students will be able to generate and analyse their own model.
- 

This module will examine how decisions involving multiple objectives can be analysed with the aid of a computer software package.

The V·I·S·A (Visual Interactive Sensitivity Analysis) software package, developed by Val Belton, provides the means for the user to develop, organise and synthesise information in a simple and transparent manner. As a decision support tool it enables the user to gain invaluable insights and provides a means to justify and explain the user's reasoning and rationale for their final decision.<sup>1</sup>

The methodology for this module is to apply the Multi-Criteria Decision Analysis (MCDA) approach and illustrate, where appropriate, how V·I·S·A can be utilised. This module will provide a step-by-step guide into how to operate the V·I·S·A program and how its many features can be employed. To achieve this, the first five of the seven steps of the Multi-criteria Decision Making process will be incorporated. These steps are:

1. Problem Identification and Formulation ☺
2. Problem Structuring ☺
3. Data ☺
4. Initial Results ☺
5. Sensitivity Analysis ☺
6. Insights, Conclusions and Recommendations
7. Go back to 1?

## Problem Identification

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### *Objectives:*

- Students will be able to identify the problem at hand and define clearly its nature and parameters.
  - Students will be able to determine if the MCDA approach and V·I·S·A would provide valuable insights to the decision maker(s).
- 

The first important step in the MCDA approach is to clearly define the problem or dilemma that faces the decision-maker(s). This may comprise a situation that requires a one-off decision or one that requires multiple, ongoing decisions.<sup>2</sup> We handle many multi-criteria problems every day, most of which are resolved intuitively. So when does a multi-criteria decision warrant the building of a model and the use of a computer-based approach? The MCDA model approach is suitable when an intuitive approach is not appropriate, for example because the decision-maker(s) feel the decision is too large and complex to handle intuitively, because it involves a number of conflicting objectives, or involves multiple stakeholders with diverse views. Often there is a desire for a formal procedure so that the decision making process can be made open and transparent, and is seen to be fair. If the problem is one that can be structured as involving a collection of alternatives that can be tested against several criteria, then the MCDA approach may be suitable.<sup>3</sup> Such alternatives may be alternative choices, actions, strategies, or alternatives units.

An inadequate or poorly defined problem will often lead to a poorly structured and ill-informed decision being made. On the flip side, a well-structured and clearly defined problem can lead to a higher level of understanding and a wealth of new information.

The decision-maker(s), in determining whether MCDA will aid in their decision-making, should look to work through the following focusing questions: (Use these for your own model)

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***Focusing Questions:***

- What is the significance of the problem or dilemma? Is it a small problem that is suited to an intuitive approach, or does it warrant a more formal approach?
- Can the problem or dilemma be broken down into key components (criteria)? What are they? Do some of the criteria appear to be in conflict with each other?
- Does the problem or dilemma involve several alternatives that need to be compared?

- 
- Are these alternatives already identified or will further research be required?
  
  - What is your pre-conceived judgement of the problem?
  
  - Would the MCDA approach and V·I·S·A decision support tool be useful in providing further insight for the final decision?

### *Example: Test Cricket*

In recent years within the cricketing world, growing debate has been documented over which of the test playing nations is the world's finest. After the completion of the 1997 Ashes cricket series between Australia and England (in which Australia won 3-2), Australian cricket captain Mark Taylor added fuel to the debate by stating that Australia is the world's premier test cricket team in the modern era. Since making this statement, many of the other test cricketing nations, combined with various cricket ambassadors, officials and personalities have either hotly refuted or supported this claim.

The question of whom is test cricket's finest team still remains largely unanswered, if only unanswered through a plausible and acceptable scoring system rather than subjective opinion. No clear system or proposal has been implemented to score each team's performance. In recent years, discussion on this issue has been centred on the development of a 'World Test Challenge'. This challenge would comprise each test cricketing nation playing each other over a two to four year period, with one test match or test series being played on a home and away basis. At the end of this round robin play, a scoring system would determine the overall winner or champion.

Would the MCDA approach, supported by V·I·S·A, provide a good basis for determining the best cricketing nation? Use the focusing questions outlined above to determine if it would.

- What is the significance of the problem or dilemma? Is it a small problem that is suited to an intuitive approach, or does it warrant a more formal approach?



- Can the problem or dilemma be broken down into a number of key components (criteria)? What are they? Do some of the criteria appear to be in conflict with each other?
- Does the problem or dilemma involve several alternatives that need to be compared?
- Are these alternatives already identified or will further research be required?
- What is your pre-conceived judgement of the problem?
- Would the MCDA approach and V·I·S·A decision support tool be useful in providing further insight for the final decision?

### ***Example: Apartments***

After completing the Victoria University MBA program, a group of four graduates secured good jobs with organisations that are based in Wellington. While discussing their jobs with each other they found they were all facing the same concerns over living arrangements. All four were able to carry out a large portion of their work at home, and this has led to all of them purchasing 'lifestyle' properties based in various locations around the lower North Island. When required in Wellington for meetings and the like, each graduate would travel down and spend a few days in Wellington living out of a hotel room. The cost of this accommodation fell on the graduates, as it was their decision to live out of the Wellington area.

Following lengthy discussion and research, the four graduates found that if they combined resources they could purchase an inner-city apartment for approximately the same cost as the expenses of a hotel room. Purchasing an apartment would enable the graduates to live in the area when they wanted - mainly when work commitments required. Furthermore, they found that their work commitments differed during the year for each person. Finally, the apartment option was seen as an advantage as it would build up equity in an asset at the same time.

Agreement was reached to look for an apartment that would present the best overall value for money and ease of use for all group members. The problem remained how to choose an apartment that would suit everyone's preferences...

Does the above scenario suggest MCDA would be appropriate? Apply the following focusing questions to decide.

- What is the significance of the problem or dilemma? Is it a small problem that is suited to an intuitive approach, or does it warrant a more formal approach?

- Can the problem or dilemma be broken down into a number of key components (criteria)? What are they? Do some of the criteria appear to be in conflict with each other?
- Does the problem or dilemma involve several alternatives that need to be compared?
- Are these alternatives already identified, and if not, could they be identified fairly readily?
- What is your pre-conceived judgement of the problem?
- Would the MCDA approach and V·I·S·A decision support tool be useful in providing further insight for the final decision?

***Answer: Test Cricket***

Yes it would. The nature of the problem suggests that a straightforward, transparent and structured approach would be required to provide a defensible answer. V•I•S•A would provide a balanced perspective, which could address the debate that has raged in cricketing circles. A range of alternatives is available - namely, the set of test cricket playing nations. In addition, multiple criteria can be developed based on data such as the outcomes of test matches and test match series in a given period of time.

***Answer: Apartments***

Yes it would. MCDA would provide a sensible and logical framework for determining the best overall apartment. A range of alternatives (possible apartments) and a set of criteria could be developed from a variety of resources ("hard" data) and from their own perspectives and intuition ("soft" data).

## Problem Structuring

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### *Objectives:*

- Students will be in a position where they can readily generate criteria and alternatives that suit the decision problem.
  - Having achieved this, students will be able to develop an appropriate hierarchical criteria tree structure for analysis.
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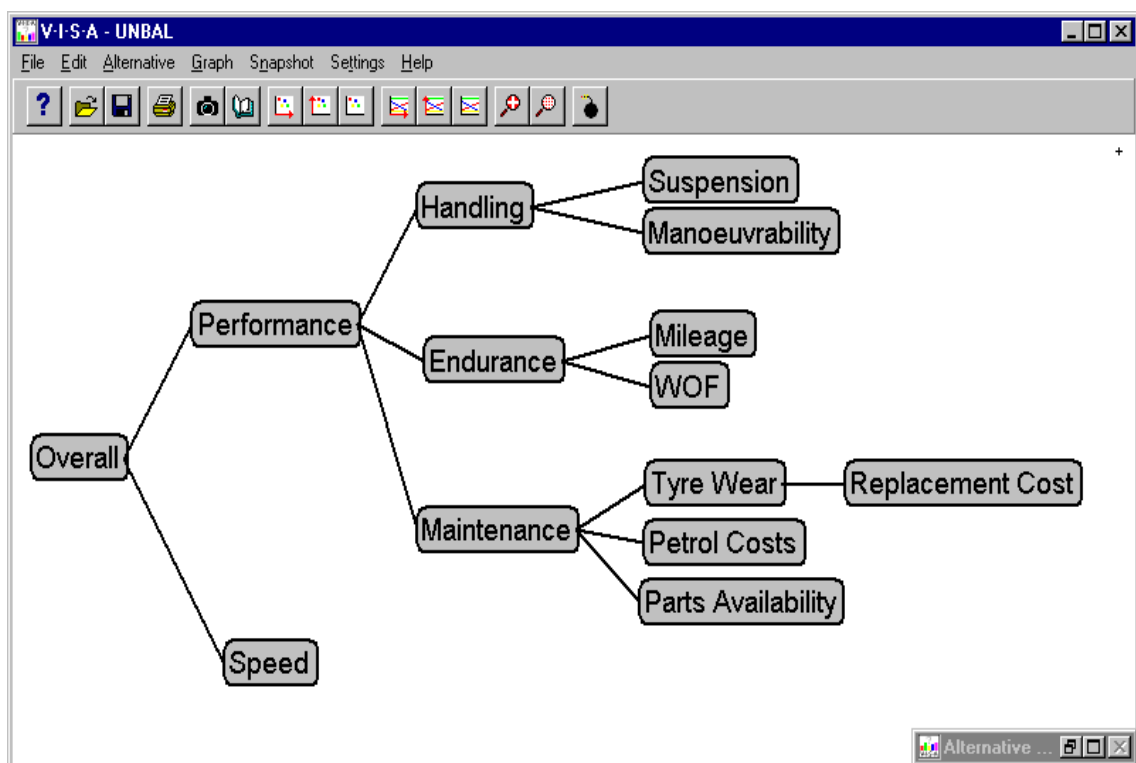
Having concluded that the MCDA approach is suitable for the decision problem at hand, the next step is to begin structuring the problem with the aim of developing a set of alternatives and a criteria tree hierarchy.

### *Identifying Criteria*

Criteria can be identified in a number of ways. The purpose of identifying criteria is to develop a hierarchical tree, (called the "Value tree") and develop the means by which the set of alternatives will be tested and compared. Brainstorming - either alone or in a group - will often elicit a wide set of criteria, and can be done whether or not the set of alternatives is already established.

- K When criteria are not so forthcoming, Val Belton<sup>4</sup> suggests other methods that can be used. If the set of alternatives is known, three useful approaches are the Plus-Minus-Interesting Points method (using 1 alternative); the Pairwise Comparisons method (comparing 2 alternatives at a time); and the Repetory Grid method (comparing 3 alternatives at a time). For top-down models, she suggests starting by defining objectives and boundaries, and also using the methods just described.
- K Some criteria, by definition, may be considered to be overriding, or pre-conditions to selecting the alternatives. These types of criteria are referred to as Pre-emptive criteria and are used to screen out alternatives before they are evaluated against the criteria on the value tree. These pre-emptive criteria are not even shown on the value tree. As an example, a pre-emptive criterion might be that only options costing between \$5,000 and \$15,000 would be considered.

- K While developing a set of criteria, it is important to be aware of the difference between differentiating and non-differentiating criteria.<sup>5</sup> When the decision to be made is one of choice or ranking of the alternatives, only differentiating criteria need be used. Non-differentiating criteria can be excluded from the value tree, as they would provide no added value to this type of decision problem, so long as the set of alternatives is not added to. For example, in comparing candidates for a job, all applicants might have similar qualifications, in which case this criterion is non-differentiating and could be omitted. However, if a new candidate is added to the list, this criterion will become relevant (differentiating) if the new candidate has quite different qualifications.
- K The level of detail within the tree hierarchy should be such that it is balanced, ie the level of detail should be appropriate and on a similar level for different branches of the tree.<sup>6</sup> An example of an unbalanced tree is:



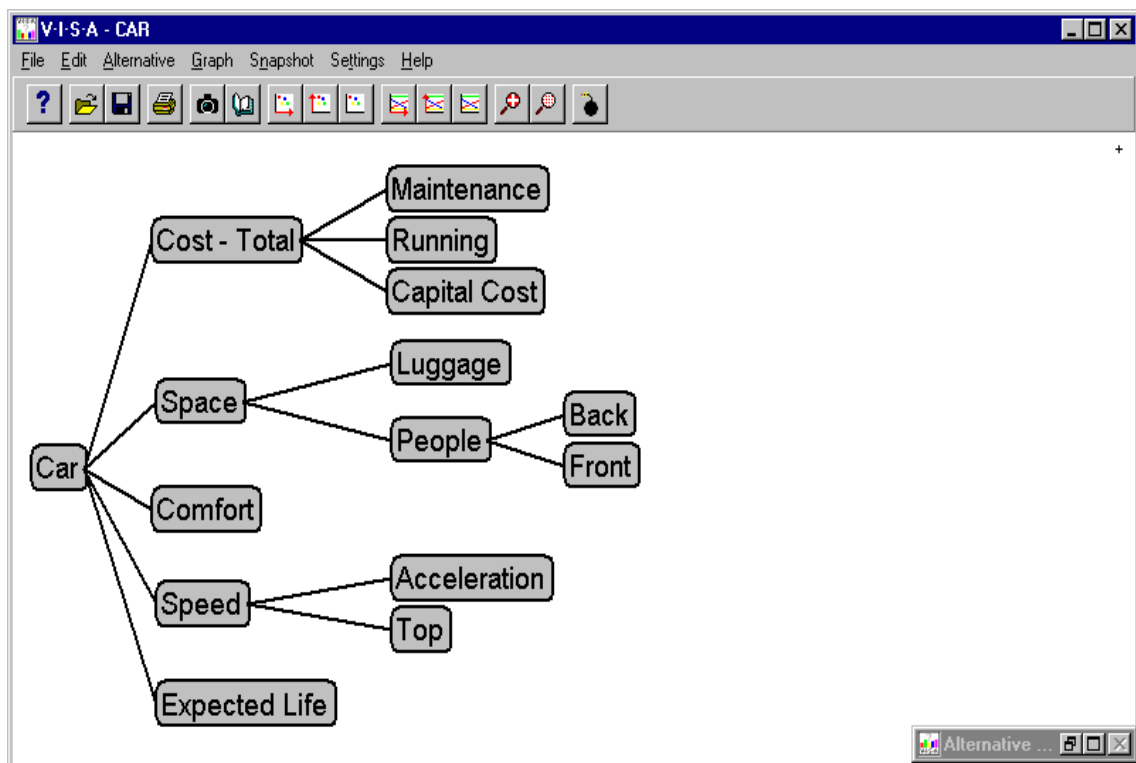
Belton also discusses other attributes of the tree.<sup>7</sup> The tree should avoid using two or more criteria that essentially measure the same attribute. This would essentially amount to double counting. If a criterion is hard to measure, the decision maker(s) may use a proxy variable. The criteria must also satisfy the condition known as preference independence.<sup>8</sup>

K To determine if the criteria hierarchy tree is appropriate, Keeney and Raiffa have suggested five attributes which the tree should possess (refer to Goodwin and Wright<sup>9</sup> for descriptions):

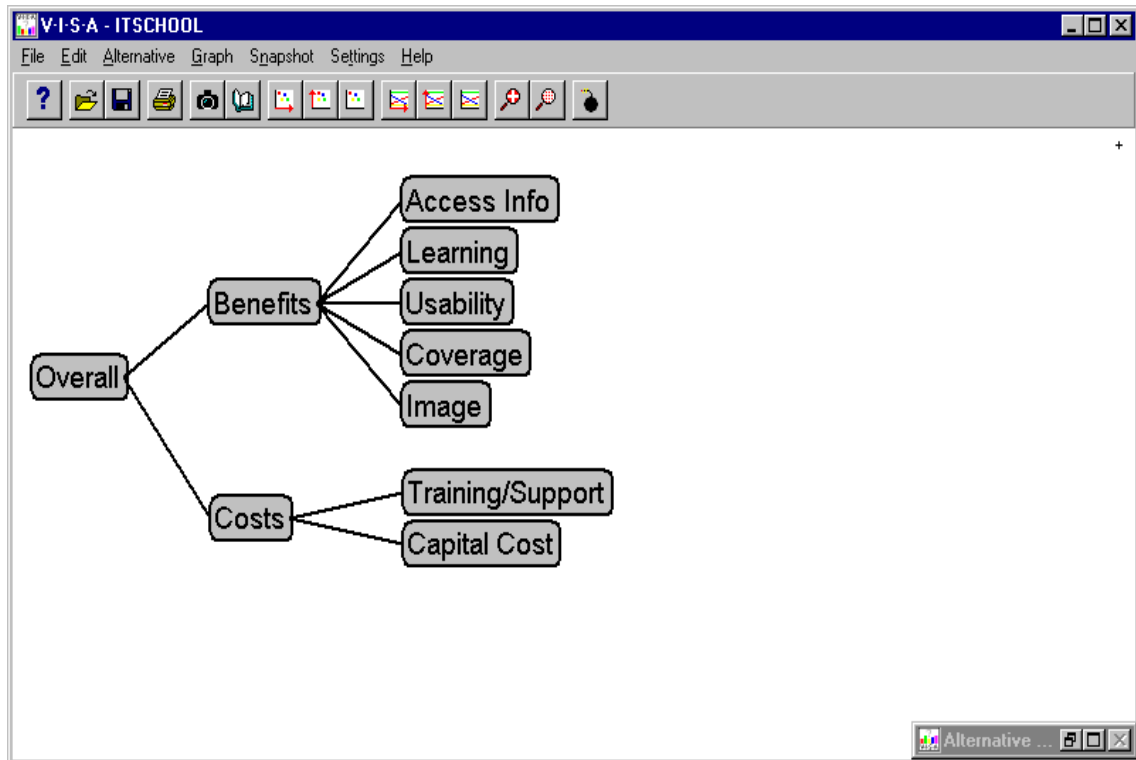
1. Complete
2. Operational
3. Decomposable
4. Absence of redundancy
5. Minimum size

Using these five attributes, determine if the following tree structures are appropriate:

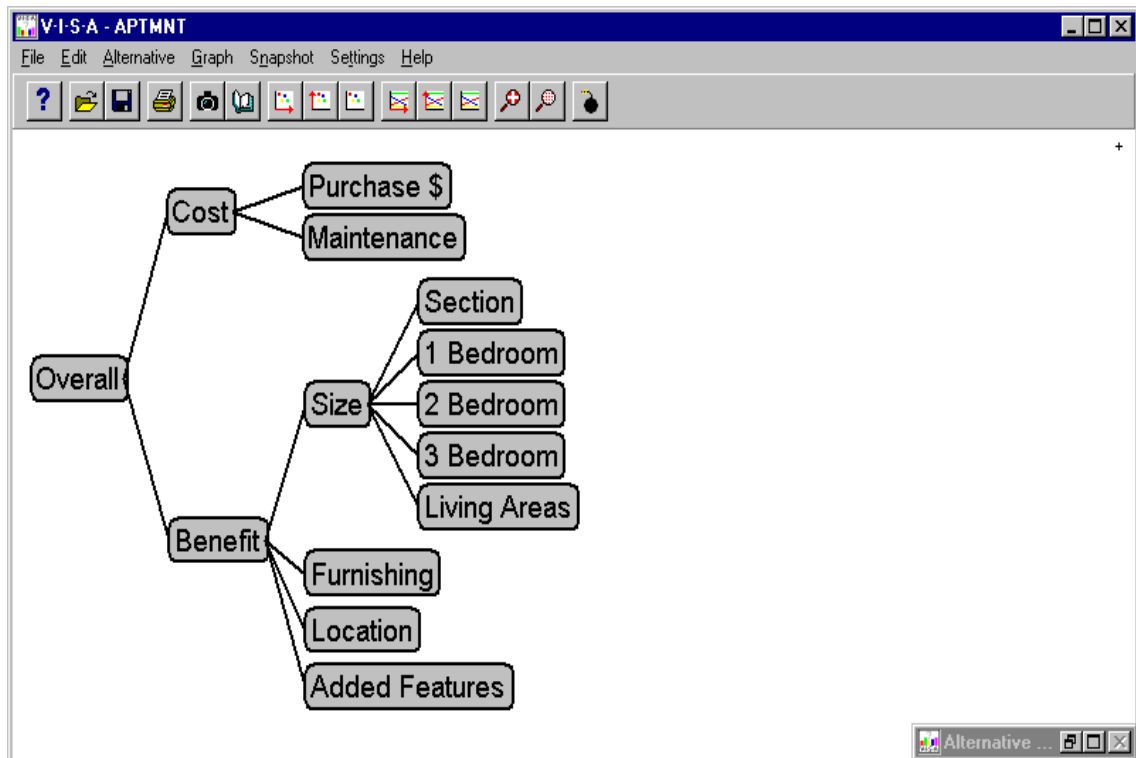
**Example 1: Choosing between cars**



**Example 2: Choosing the best information technology package for a school.**



**Example 3: Purchase of a house**





***Answer: Choosing between cars***

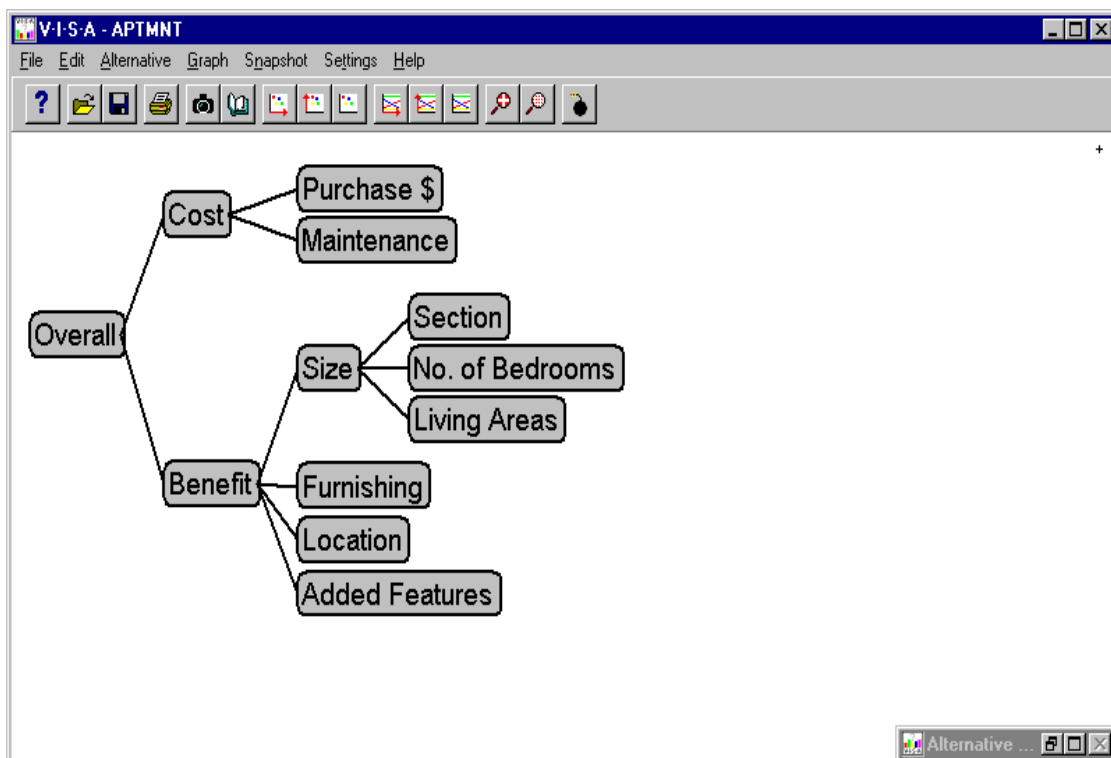
Yes, the hierarchy appears to be suitable enough and should enable the decision maker(s) to determine the best car to purchase, although they may wish to add a criterion for Style or Colour. Often we overlook these important, but supposedly emotional criteria, in a formal model - yet if they influence our preferences, they should be included. Otherwise the model results won't gain the acceptance of the decision-makers.

***Answer: Choosing the best IT package for a school.***

Again this would a suitable criteria hierarchy for the decision maker(s) if it covers all relevant criteria. It should be noted that with any cost/benefit analysis, either the costs and benefits would need to be measured in some time scales, or an account of this time scale should be taken in the final weightings. This model was actually used in a real world situation.

***Answer: Purchase of a house***

This hierarchy is slightly incorrect. Under the Size branch the decision maker(s) have opted to distinguish between 1, 2 and 3 bedroom places. This distinction would be more appropriate to make in the scoring. This will become clearer later on. The decision maker(s) would be better to eliminate the 1, 2 and 3 bedroom criteria and replace them with a single criterion - Number of bedrooms, as shown below:



For your own example, use the following focusing questions to assist you in drawing the tree for your decision.

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***Focusing Questions:***

- Do criteria exist which can be used to evaluate the differences in alternatives? What are they? Think of all the factors that influence your decision, including subjective factors. Use the 4 methods described earlier, namely brainstorming, plus-minus-interesting points, pairwise comparisons, and repertory grid.

- Are some of these criteria pre-emptive? Can they be used to screen out alternatives? What are they?

- 
- Will all the criteria help to assess the alternatives? Are some of the criteria non-differentiating? If so, should they be excluded?
  
  - Does the situation or problem suit an Input/Output analysis, or Cost/Benefit analysis, or a more general tree structure analysis (with more than 2 first-level criteria)? For the appropriate tree structure, group your criteria under the appropriate first-level criteria.
  
  - Do the selected criteria fall into a logical hierarchical sequence? Draw it. Keep in mind the goal or objective of the decision - and use this to keep your tree as small and focused as possible. For an Input/Output or Cost/Benefit model, use these two top headings as the two top-level criteria in your tree. (Refer back to the School IT example, or the House purchase example for examples of cost/benefit models.)

## *Identifying Alternatives*

Through identifying the problem at hand it will often become clear what alternatives are available. However, having identified these alternatives it is necessary to investigate whether any alternatives have not been recognised or are 'hidden' to the decision-maker(s). Often further research or group discussion will elicit further alternatives.

The aim of the decision-maker(s) should be to develop a set of alternatives that is representative of all the options available. To reduce a larger set, the decision-maker(s) can use a screening process. The final set of alternatives should be appropriate, representative and manageable while covering all the possibilities.

Within this screening process, it is important to ensure that there exist no overlapping or joint alternatives. The best set usually incorporate alternatives which are independent (mutually exclusive) of each other, or ones which are loosely dependant but not entirely dependent. Later, when we start scoring the alternatives, it may become apparent that an alternative is dominated by another alternative (in that the alternative is not preferred to the second alternative on any criteria), in which case the dominated alternative can be dropped from consideration.

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### *Focusing Questions:*

- Can alternatives be readily recognised? What are they?
  
  
  
  
  
  
  
  
  
  
- Do they suit the decision problem, i.e. will they help the decision maker(s) achieve their goal?

- 
- Will comparing these alternatives elicit the right answers for the problem or dilemma?
  
  - Do other significant alternatives possibly exist that are unknown? Can these be found? How?
  
  - Do any alternatives overlap with others or are they joint alternatives? If so, how can they be grouped or redefined so that all alternatives are mutually exclusive?
  
  - Can alternatives be excluded because they are dominated by other alternatives?
-

### ***Example: Holiday***

Once upon a time, there lived a man called Brendan. Brendan was enjoying Wellington's summer sunshine as he lay next to his swimming pool. His girlfriend was swimming effortlessly in the pool, allowing Brendan the opportunity to daydream a little.

Brendan was in a situation where money was of no importance to him. Yet he felt that his laid back lifestyle was in need of a pick-me-up. And so he began dreaming of an overseas holiday ... but it had to be a holiday with a difference.

Brendan envisaged a holiday that had excitement, adventure but also had times where he could relax and see the sights - both old and new. There were also the more mundane considerations to take into account such as transport and accommodation.

As the hours passed by, Brendan had flashes of the surf in Hawaii, the islands of Greece, the pyramids of Egypt, the castles of Scotland, the pizza of Italy, the beaches of Fiji, the temples of Thailand, and the mountains of Tibet. But which one to choose?

...And more importantly, should he take his girlfriend...

If Brendan applied the MCDA approach to his problem, what criteria should he use and what might the hierarchical tree structure look like?

What alternatives does he have?

***Example: Apartments***

After deciding that their problem warranted the use of MCDA and V·I·S·A, the four graduates sat down and analysed their situation. They decided that the situation at hand required them to use a number of pre-emptive criteria to screen out alternatives. These were:

- That the apartment should not exceed \$250,000 in price.
- That the apartment had to be in the inner-city area and no further than 10 minutes walking distance from the furthest place of work.

After individually brainstorming for criteria, the four graduates arranged to meet together and decide on an initial set of criteria that would enable them to look for alternatives that met these initial criteria. The group considered the following factors were desirable in an apartment:

- Had a balcony to enjoy Wellington's beautiful weather and so the wide doors leading out to the balcony would allow fresh air to circulate more quickly through the apartment. This was important especially when the apartment went unused for long periods of time.
- Had one or two bedrooms.
- Was fully or semi-furnished as the apartment needed to be used straight away. Also provided ease of use.
- Had morning and/or afternoon sun both for intrinsic reasons (mood) and for the welcoming nature of the apartment. Afternoon sun was seen as more important as the graduates would be at work more often in the morning.
- Had amenities close by.

- Had a small number of tenants (preferably no greater than five) so that the apartment building remained quiet especially when they wanted to relax.
- Had parking as it adds value to the apartment and two of the four graduates drive. A car parking building that was within two minutes walk was felt to be another option to consider.
- Should have elevator access
- Be of solid construction and provide reasonable levels of security

What criteria should be used in the criteria tree? How should the criteria tree be structured? Use the focusing questions to determine a suitable criteria tree.

- Do criteria exist which can be used to evaluate the differences in alternatives? What are they? Think of all the factors that influence your decision, including subjective factors. Use the 4 methods described earlier, namely brainstorming, plus-minus-interesting points, pairwise comparisons, and repertory grid.

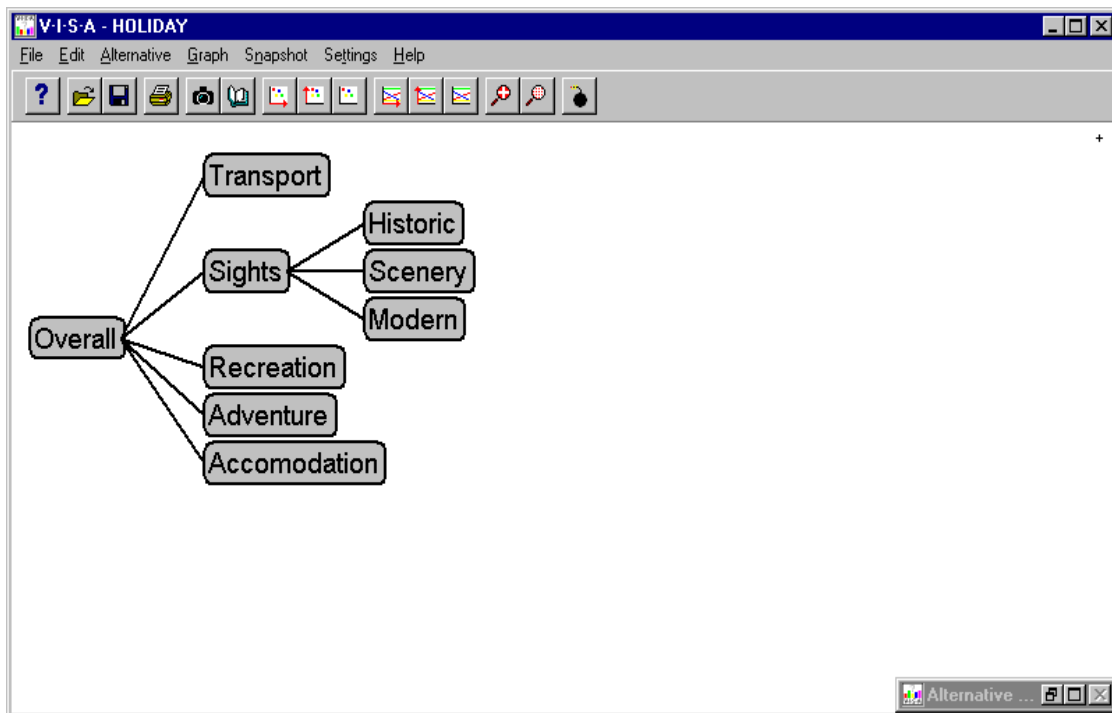
- Are some of these criteria pre-emptive? Can they be used to screen out alternatives? What are they?



- Will all the criteria help to assess the alternatives? Are some of the criteria non-differentiating? If so, should they be excluded?
  
- Does the situation or problem suit an Input/Output analysis, or Cost/Benefit analysis, or a more general tree structure analysis (with more than 2 first-level criteria)? For the appropriate tree structure, group your criteria under the appropriate first-level criteria.
  
- Do the selected criteria fall into a logical hierarchical sequence? Draw it. Keep in mind the goal or objective of the decision - and use this to keep your tree as small and focused as possible. For an Input/Output or Cost/Benefit model, use these two top headings as the two top-level criteria in your tree. (Refer back to the School IT example, or the House purchase example for examples of cost/benefit models.)

## Answer: Holiday

When most of us go on holiday, cost is an important consideration, and so a Cost/Benefit model would often be appropriate. But for Brendan, cost is of no importance, so a Cost/Benefit analysis is inappropriate. Instead it is better to focus on the benefits that each option provides. From the information provided, the following tree structure was developed:



Note: that this criteria tree is not the only possible outcome. Variations of this are plausible, eg sights, adventure and relaxation may be 3 sub-criteria of Recreation. Secondly, the sub-criteria of Sights could be dropped, and taken into account when scoring this criterion.

Brendan's set of alternatives include:

- Hawaii
- Greece
- Egypt
- Scotland
- Italy
- Fiji
- Thailand
- Tibet

### ***Answer: Apartments***

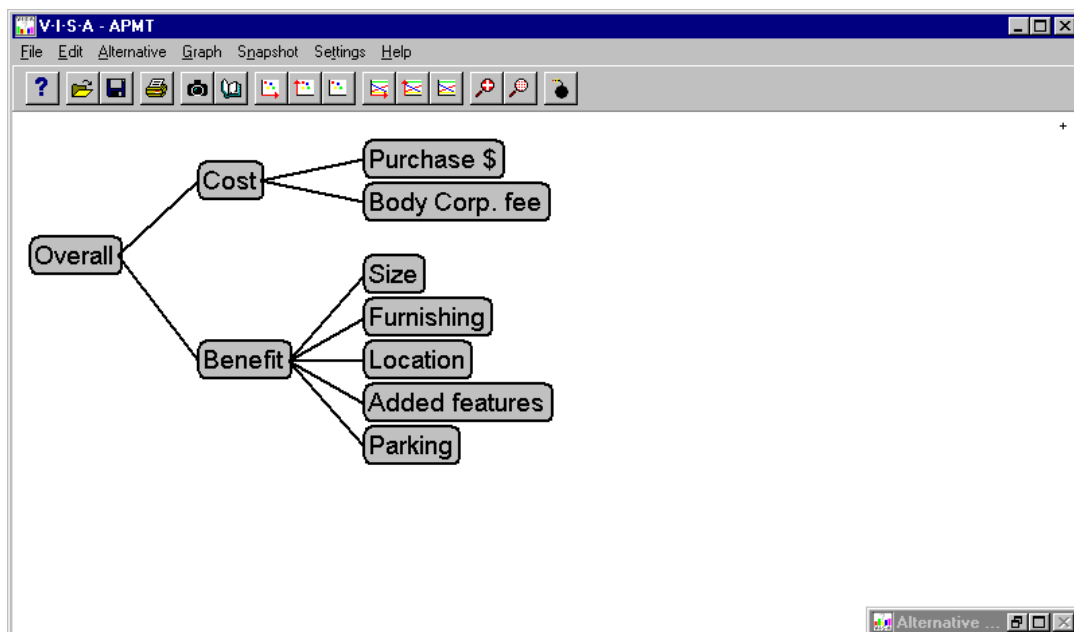
Through applying a screening process, two parent criteria and seven child criteria were selected for analysis. Ideally, the four graduates saw the problem as a cost/benefit analysis. Subsequently the parent criteria were Cost and Benefit. The sub-criteria, or child criteria, under the Cost criterion were selected as:

- The Purchase price of the apartment (*Purchase \$*)
- The Fee incurred through using an agency (*Body Corp. fee*).

The sub-criteria under the Benefit criterion were selected as:

- The *Size* of the apartment (square meters).
- *Furnishing* - how well the apartment is furnished: eg fully furnished, semi furnished or unfurnished.
- The *Location* of the apartment relative to each graduate's place of work.
- *Added features* - balcony, sun, number of tenants, amenities close by.
- *Parking* - Access and convenience of parking, ranging from on-site parking, car parking building nearby (2 minutes walk) to no parking.

Using the above criteria, develop the criteria tree and save the V·I·S·A model as 'Apmnt'. You will continue to develop this model through the rest of this module.



## *Objectives:*

- Students will be able to perform simple tasks within the V·I·S·A decision support tool, including: opening an existing model, starting a new model, developing a tree structure and entering alternatives.

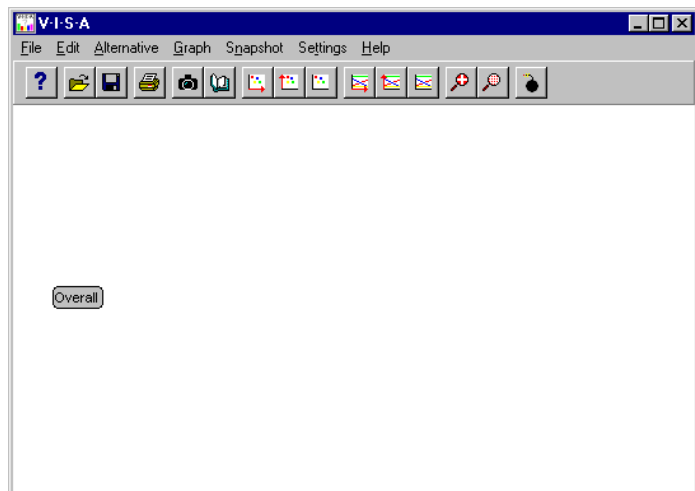
Before explaining the procedure for assigning scores and weights to the decision model, it is perhaps more meaningful to begin using the V·I·S·A decision support tool and explain its various features.

## K **Opening V·I·S·A**

To open the V·I·S·A software program, left-click on the Start menu at the bottom left of the computer screen and select Programs/VISA/VISA. This will open the V·I·S·A program.

To open an existing V·I·S·A model, choose the File/Open pull-down menu option at the top-left of the screen.

To begin a new V·I·S·A model, right-click anywhere on the main window. This will create an Overall criterion, which will be used as the root of the hierarchy. The best location for the Overall criterion is in the middle of the main window at the left-hand side of the screen. This will enable the tree, when constructed, to flow from left to right. To move the Overall criterion, simply place the pointer on the grey box, left-click and hold, then drag the box to its desired location.

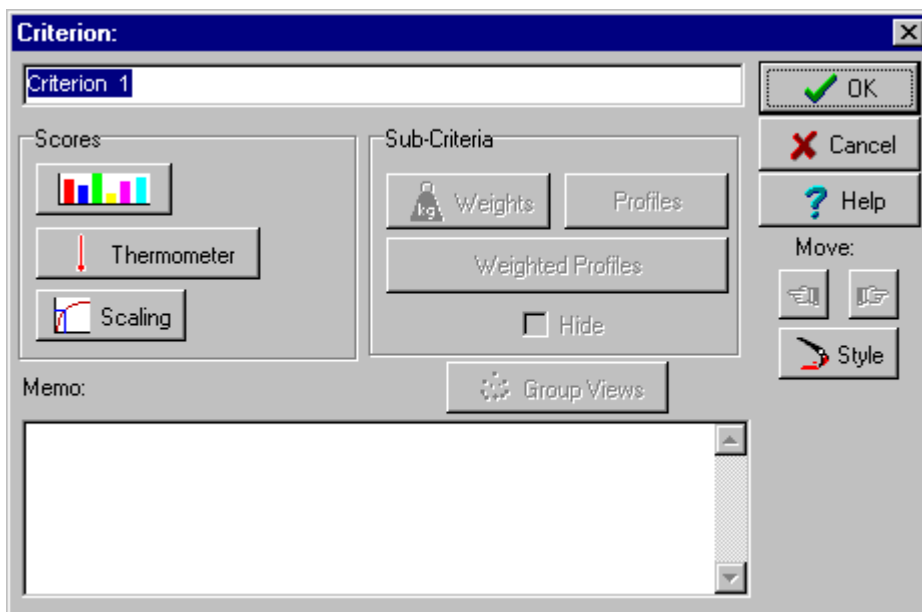


V·I·S·A also has its own demo models and tutorial help that you can access on opening V·I·S·A.

## K Tree Hierarchy

To construct the hierarchical tree, simply right-click anywhere on the main window. A new criterion will be created which will be attached to the Overall criterion. Each successive criterion that is created will also attach itself to the Overall criterion. To link two criteria together to form a parent/child relationship, right-click and hold on the child criterion, and drag the cursor to the parent criterion, then release.

To rename the criteria, simply left click on each criterion box and type the desired name in the dialog box. Further information can also be inputted into this dialog box as desired.

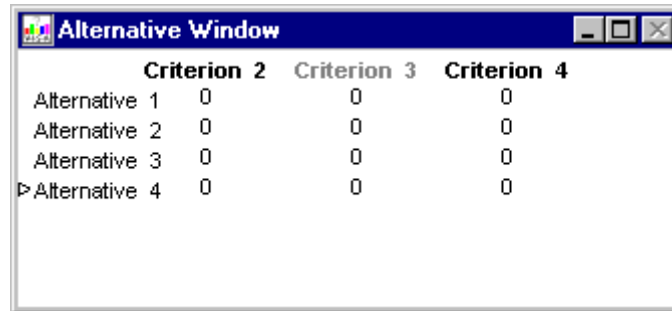


To delete a criterion, click on the Bomb icon in the toolbar, and place the icon on the criterion to be deleted. A note of warning that any child criteria attached to that criterion will also be deleted.

When you have finished your tree it is useful to tidy the tree so that the links between criteria are easily observed. V.I.S.A. has a Tidy option available under the Edit pull-down menu option. After completing this, you can magnify (Zoom In) your tree by selecting the magnifying glass (plus) option on the main icon menu.

### *K Entering the Alternatives*

The alternatives can be inputted in the Alternatives window by clicking on the maximise , or enlarge  button at the top right of the Alternatives window. To include a new alternative, choose Alternative/Add from the main menu bar. Alternatives can also be added by using the shortcut key Ctrl A. To change the name of each alternative, click on the alternative to be changed and a dialog box will appear.



The screenshot shows a window titled "Alternative Window" with a table containing the following data:

	Criterion 2	Criterion 3	Criterion 4
Alternative 1	0	0	0
Alternative 2	0	0	0
Alternative 3	0	0	0
▶ Alternative 4	0	0	0

### ***Example: Apartments***

Having identified the pre-emptive criteria and the list of desired criteria, the group went out, researched and developed a set of apartment alternatives that met the pre-emptive criteria, and looked good against their desired criteria. In all ten apartments were considered:

**City 1** - One bedroom apartment situated on the fringe of the CBD. Your own private balcony is perfect for enjoying breakfast in the sun. A secure car park adds to the attraction of this unit. \$166,000. Body Corporate (BC) fee \$1,400.

**City 2** - Two-bedroom apartment located handy to the central city, afternoon sun. \$146,000. BC \$1,200.

**City 3** - This two-bedroom apartment has a balcony, which gets good morning sun, and is also semi-furnished. \$227,000. BC \$2,000.

**City 4** - Prime location! One bedroom apartment situated perfectly for the busy executive. Semi-furnished with all whitewear. \$182,000. BC \$2,000.

**City 5** - Avoid the traffic hassles. One bedroom apartment enjoying afternoon sun. \$136,000. BC \$1,000.

**City 6** - This one bedroom apartment is situated in a top location. Good morning sun. \$142,000. BC \$1,150.

**City 7** - Just move in! This two-bedroom apartment is situated handily to the CBD and is fully furnished. Secure car parking and a great balcony are just two of the additional features. \$250,000. BC \$2,000.

**City 8** - Well situated, this two-bedroom apartment has a car park, is semi-furnished, and enjoys great all day sunshine. \$225,000. BC \$1,800.

**City 9** - Two-bedroom apartment located on the fringe of the city. Secure car park and a balcony are two of its features. Semi-furnished is another bonus. \$250,000. BC \$1,750.

**City 10** - Enjoy the afternoon sun from your balcony when you buy this two-bedroom apartment. Good location. \$146,000. BC \$1,300.

After visiting each apartment the group were able to apply the other methods (plus-minus-interesting points technique, pairwise comparisons, and repertory grid) for eliciting criteria to check whether they had missed anything important. They were also able to eliminate potential criteria that were inappropriate or non-differentiating. The non-differentiating criteria included elevator access, the type of construction used, and security

requirements. In eliminating these non-differentiating criteria, the group was mindful that these criteria might require including in the model if any alternatives were added later which did not meet these requirements.

They were now ready to begin using the V·I·S·A program....

- K Using the 'Apmnt' model you developed earlier, open the Alternatives window and create 10 alternatives. Rename each alternative City 1, City 2, City 3, ..., City 9, and City 10. Save your model again.

	Purchase \$	Body Corp. fee	Size	Furnishing	Location	Added features	Parking
City 1	0	0	0	0	0	0	0
City 2	0	0	0	0	0	0	0
City 3	0	0	0	0	0	0	0
City 4	0	0	0	0	0	0	0
City 5	0	0	0	0	0	0	0
City 6	0	0	0	0	0	0	0
City 7	0	0	0	0	0	0	0
City 8	0	0	0	0	0	0	0
City 9	0	0	0	0	0	0	0
City 10	0	0	0	0	0	0	0



---

### *Objectives:*

- Students will be able to develop a suitable scoring system and justify the system used.
  - Students will be able to understand the differences between a global and local scoring system; linear and non-linear value function; and, when scoring should be reversed.
  - Students will be able to develop an appropriate set of weights that reflect the scoring systems used, their own preferences, and the type of decision required.
- 

### *Scoring*

At this stage, the decision-maker(s) should be in a position where they have a sufficient understanding of the problem and its components, that they feel ready to define a scoring system to evaluate the performance of each alternative against the criteria. Scoring refers to the assignment of values to each alternative. This enables the decision-maker(s) to gauge and reflect the importance, or significance, of each alternative under each child criterion.

K This type of MCDA assumes that alternatives are scored by decision-maker(s) using an interval scale.<sup>10</sup> The V·I·S·A software package supports a 0 to 100 scale for scoring. There are two ways of defining the end-points on the scales: global and/or local interval scales. Val Belton provides a good explanation of the difference between local and global scales.<sup>11</sup> In the explanation of the difference between local and global scales, Val Belton also highlights the advantages and disadvantages of using either scoring option.

K When deciding on the type of scale to use, it is necessary to consider the type of data being used. If quantitative data are being used, then either global or local scales could be used effectively. If qualitative data is being used then a local scale may be the simpler option; alternatively you can use V·I·S·A's qualitative scoring facility.

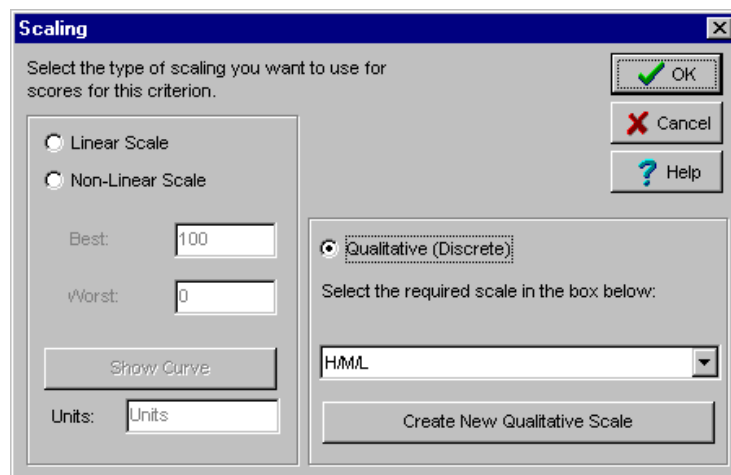
It is important to choose a method of scoring that you feel comfortable with, and which makes most sense to you. This will make it easier when assigning weights to the criteria (described later), as these weights will depend partly on the scoring scales used. It is probably easier to use either all local, or all global scales, rather than a mixture of local and global scores. Global scales are probably easier to work with for weighting, though they require more thought than the local scales at the scoring stage.

K Goodwin and Wright<sup>12</sup> provide an extensive description of two methods that can be used in determining scores - Direct Rating and Value Functions.

Selecting an appropriate qualitative scoring system can be difficult, and fortunately V·I·S·A provides some support for this. Specifically, V·I·S·A allows the user to select a built-in qualitative scoring system or to develop their own.

K A qualitative scoring system can be chosen by left clicking on the appropriate end-criterion. In the dialog box that appears select the *Scaling* icon option.

On the right of the dialog box that appears is an option that states "Qualitative (Discrete)". Select this option. Next, either select one of V·I·S·A's built-in scoring systems (H/M/L or Five

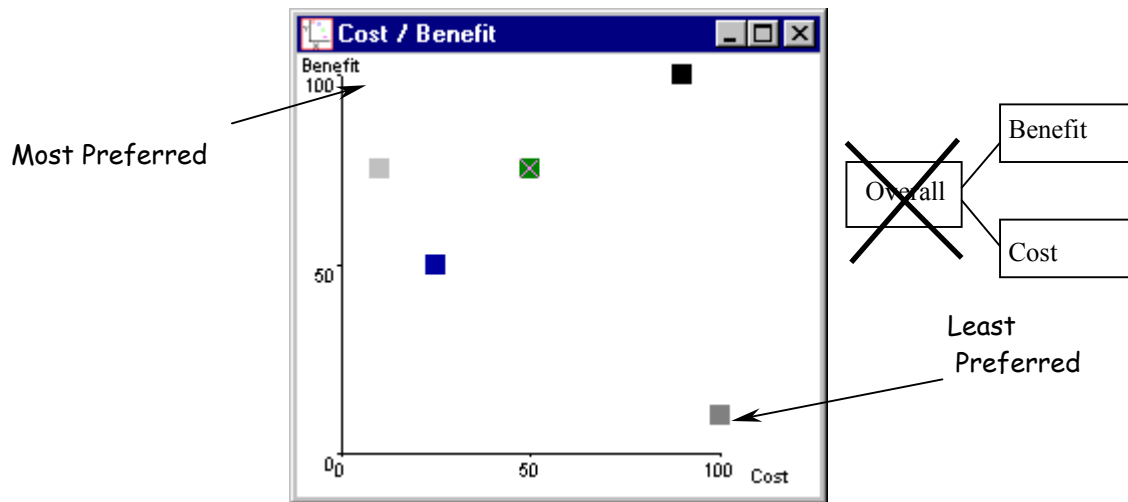


Point Scale) in the pull-down menu or select the *Qualitative Scale Details* option to develop your own scoring system. Once you have chosen the scale to use, Close the dialog box. In the alternatives window, you will notice that the scores for each alternative under the criterion you have selected the qualitative scoring system for, have changed to words (eg. Low, Medium, or High). You can change an alternative's score by left clicking on that alternative's score under the criterion. A small box will appear and you can select the appropriate score.

- K It is usual to define scales such that "more" is "best", so that the biggest (best) value is assigned a value of 100, and the smallest (worst) alternative is assigned a value of 0. However in some cases, this scale may seem contrived, for example, in situations where "more" is actually "worse". Cost and noise are typical examples of factors for which more is worse. Another example arises when scoring the performances of a sports team e.g. a high number of penalties awarded against a team should be given a low score. In these cases, adhering to the usual definition of Best = 100, you would have to say that the higher the (noise) level, the lower the score, which may go against the decision-maker's intuition, in that the scale seems to be reversed. Decision-makers may prefer to use a more natural scale in which "more" is still given a "high" VISA score even though it is "worse".

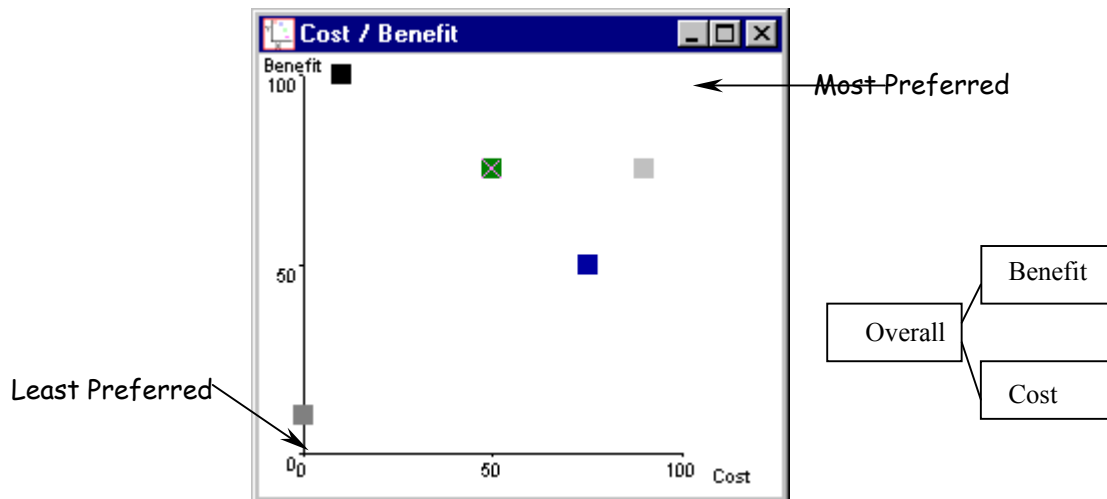
Whether to reverse the scales or not depends on the decision maker's preference, and on the type of model used. If a cost-benefit type model is used, it is a matter of preference. But if the model is of a type where all criteria are to be combined into an overall criterion, then **all criteria** must use the **same** direction of scale (normally one in which **Best = 100 and Worst =0**). Conversely if the model contains some criteria for which 100 is Best and some for which 100 is Worst, then the Overall criterion should not be used, and the two types of criteria should be separated.

In particular, if the scale is *not* reversed (eg. High cost = High V·I·S·A score) then the Overall score becomes inappropriate to use and the most desired option is found by the alternative that has the highest benefits score and *lowest* cost score.



*In the Figure above the Cost has not been reversed and the best alternative is found to be the alternative with a Benefit score of 100 and a Cost score of 0 (top left alternative of the X-Y plot)*

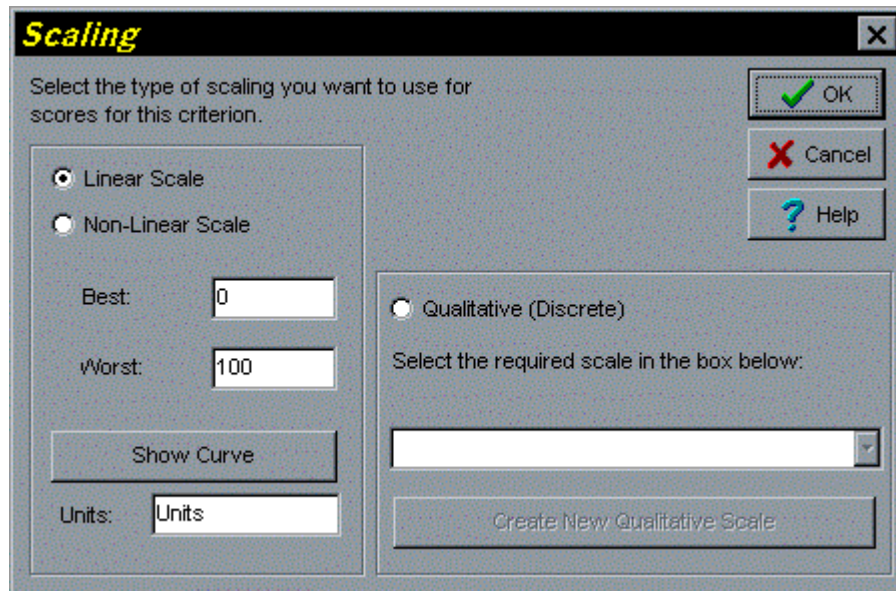
In contrast, if the scale *is* reversed (eg. High cost = Low V.I.S.A score) then the costs and benefits can be combined into an overall value criterion and the Overall score becomes appropriate. In addition, the most desired alternative is found by the alternative that has the highest benefits score and *highest* cost score.



*In the Figure above the Cost has been reversed and the best alternative is found to be the alternative with a Benefit score of 100 and a Cost score of 100 (top right alternative of the X-Y plot)*

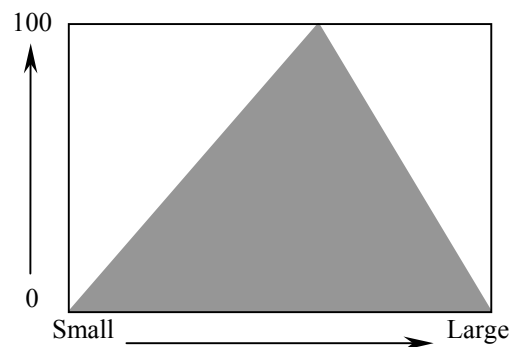
Scales may be reversed directly when scoring, or they can be reversed using a feature in V·I·S·A called scaling.

- K To reverse an end-criterion's scores using scaling, firstly left click on the end-criterion. Within the dialog box that appears select the *Scaling* icon. In the second dialog box that appears, change the "Best" dialog box from 100 to 0 and the "Worst" dialog box from 0 to 100.



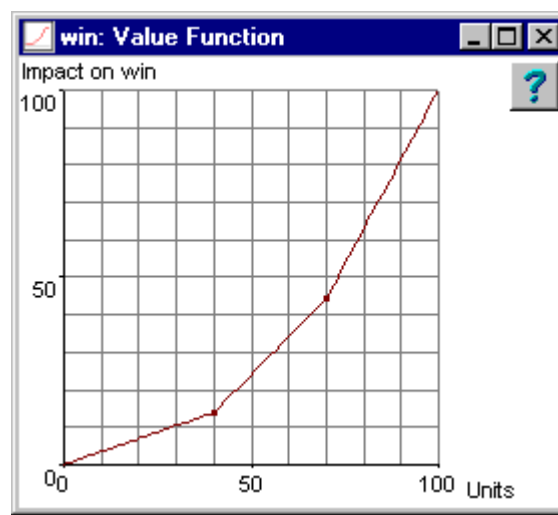
- K Other uses of the scaling feature...

Using Value Functions is helpful when you cannot measure a criterion directly, and need to resort to using a proxy variable, or when you want to make explicit a non-linear scoring scale. For example, the ideal house size will not usually be the largest or smallest house, but rather a mid-sized one. In such cases, the most preferred value occurs in the middle of the range of measured values (such as floor area), not the lowest or highest values. We can represent these using non-linear scales.



A linear scale assumes that the decision-maker's preferences increase linearly with the score on a criterion. In contrast, a non-linear scale can be used to reflect different patterns of preferences. For instance, in the sports team example, a non-linear scale may be used to reflect the differences between a high percentage of game wins over a low percentage of game wins.

This thinking may lead the decision maker(s) to change the 'Win' criteria in their model to a non-linear scale such as this:



- K By default, the scales used in the V·I·S·A package are linear. To change to a non-linear scale simply click on the end criterion to be changed, select *Scaling* then click on *Non-Linear Scale*. Next select *Show Curve* and left-click on the line and drag to re-shape the line to the desired shape.
- K If scaling is used, and/or if scales are reversed, it is **strongly advisable** to view the *Profiles chart* to check the effect on the criteria scores. Check that the high and low scores are as you expect them to be. Make sure you write it down, so you don't inadvertently interpret the scales incorrectly later.



- 
- Does the data fit in well with the chosen criteria? Why?
  
  - Does the available data appear to suit local or global interval scales? Why?
  
  - Does the situation require reversing a few scales? Why? Which criteria?
  
  
  - Should non-linear scales be used in some cases? If so, what should they be?



- 
- Can each of the criteria be scored on its own, without having to consider any of the other criteria? If not, the criteria may not be “preference independent” (see Goodwin and Wright), and the tree should be restructured or the criteria redefined. Often it suggests that the interdependent criteria should be linked in some way, such as a parent-child relationship.

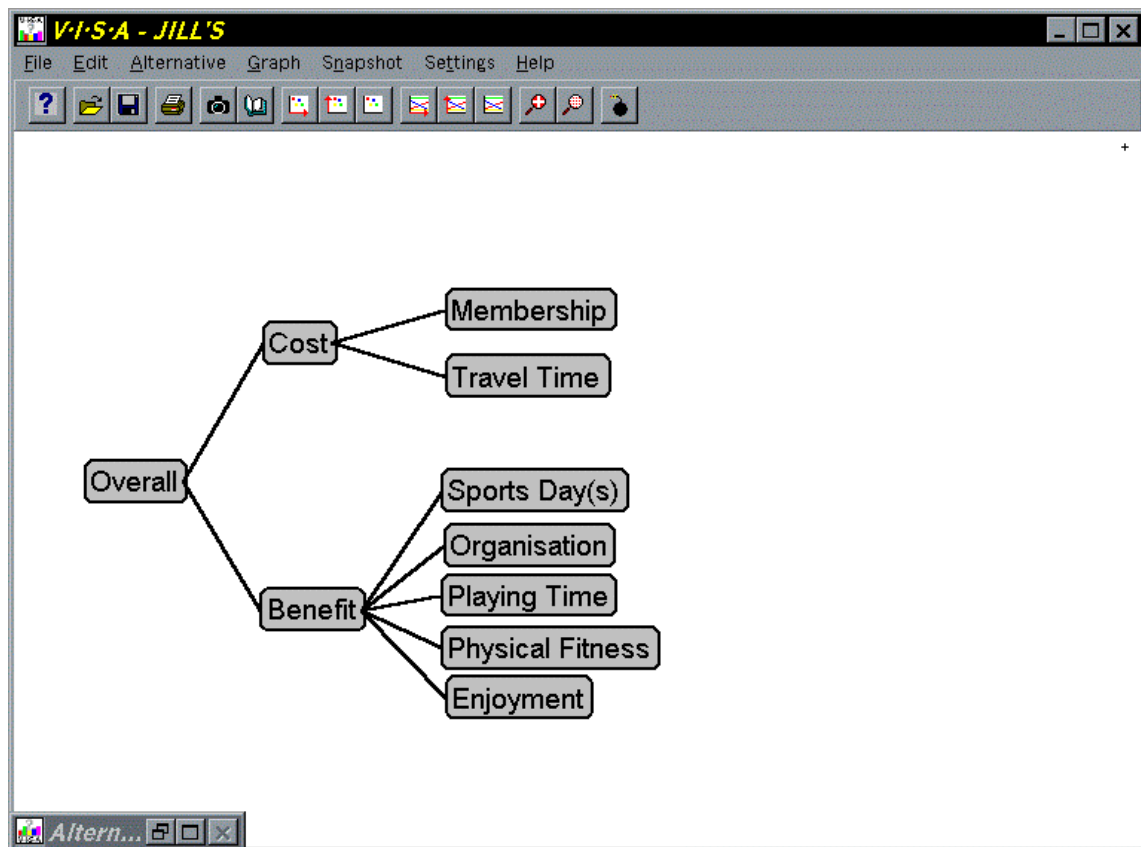
### *Example: Jill*

Due to her previous work commitments, Jill had been forced to give up her active sporting lifestyle. Now, due to a new position within a new company she has found she has some spare time. Ideally, Jill determined that she could set aside at least one night per week and at least one day in the weekend, with two days per week being preferable over only one. As the beginning of the winter season was approaching, Jill decided to reactivate her sporting lifestyle by playing a winter sport. Due to her variable work commitments, she decided she couldn't realistically commit herself to a competitive sporting team. This meant she had to find a social sport. Furthermore, as she had just started her new job, Jill decided it would be nice to get to know her associates better outside of the work environment. Finally, Jill ideally wanted to play a sport where each game lasted at least 40 minutes and required a high level of physical exertion so she could maintain her fitness levels.

Having no preference for which sport to play, Jill began scouting around for different sports clubs in her area. This involved talking to her business associates and looking through newspapers and other local newsletters. Eventually, she came up with the following alternative sports:

- Netball
- Volleyball
- Rowing
- Soccer
- Indoor Netball

To determine which was the best alternative, Jill decided to use V·I·S·A and the MCDA approach. After much thought, she came up with the following criteria tree structure:



She included an Organisation criterion as a reflection of whether her work-associates would play in the team also. Sports Day(s) captured both the convenience for her schedule and the weekly frequency of the sport.

Jill collected the following data on each of the alternative sports but was unsure how to score them on an interval scale.

	Member-ship	Travel time	Sport Day(s)	Enjoyment	Physical fitness	Organ-isation	Playing time per game
Netball	\$35	15 mins	Tuesday	Moderate	High	5 in team	40 mins
Rowing	\$50	45 mins	Wed + Sat	Low	Medium	3 in team	60 mins
Volleyball	\$15	15 mins	Sunday	Low	Low	1 in team	60 mins
Soccer	\$30	30 mins	Wed + Sat	Moderate	High	None	90 mins
Indoor Netball	\$60	10 mins	Tuesday	High	Medium	Company team	40 mins

Within V·I·S·A develop the above criteria tree. Determine an appropriate interval scoring scale for each of the above criteria and calculate the scores. Input the scores into V·I·S·A. Save it as Jill's Sport model.

***Example: Apartments***

The graduates were now at an interesting stage. They had developed their hierarchy within V·I·S·A and were now ready to input the scores. Through analysing the raw data, the graduates determined how each of the criteria would be scored and tried to classify them as global or local scoring scales. These were:

- Purchase \$ - the alternative price divided by the highest price then multiplied by 100. (Global-local scale)
- Body Corp fee - the alternative fee divided by the highest fee then multiplied by 100. (Global-local scale)
- Size - (alternative size - smallest size) / (largest size - smallest size). (Local scale)
- Furnishing - None 0, Semi 50, Full 100. (Local scale)
- Location - Fringe 0, Inner 50, Central 100. (Local scale)
- Added features - Balcony +35, Morning sun +15, Afternoon sun +25, Tenants (<5) +10, Amenities close by +15. (Global scale)
- Parking - none 0, public 50, on-site 100. (Local scale)

*The scales for Purchase price and Body Corporate fees are a mixture of global and local scales. The maximum is set by the maximum figure, but the lowest goes all the way to zero. If another apartment is chosen that is more expensive, they'll need to reset the scales, so it's probably more of a local scale than global. Furnishing etc is defined as "local" because they're based on the range of options being considered, rather than those available in the whole apartments market.*

Do you think the above scoring systems are suitable and representative?

Do they differ from your own? Why? Suggest any changes you might make and why you would make these changes.

For now we will leave the scoring system devised by the graduates as is. Using the scoring systems provided above, calculate the scores and enter them into the appropriate cells in the spreadsheet on the next page. The raw data is also provided on this page. Then open a new Excel workbook and copy the scores onto the Excel spreadsheet (Remember to follow the instructions given on page 36 carefully: cell A1 must be blank, Row 1 should contain the criteria headings, and column A the alternatives' names, with data starting in cell B2.) Save the new spreadsheet as 'ApmntScores' and save as a .txt (text tab delimited) file. Close the Excel file.

Open your Apartment model within V·I·S·A and import the ApmntScores.txt data file through the *File/Import/EXCEL TXT file* pull down menu option. Check the scores appear in the Alternatives widow. Save the Apartment model.

**Original Data**

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1	\$166,000	\$1,400	175.0	No	Fringe	Bal,Mor,Ame	On-site
City 2	\$146,000	\$1,200	190.0	No	Inner	Aft,Ame	Public
City 3	\$227,000	\$2,000	187.5	Semi	Fringe	Bal,Ten	None
City 4	\$182,000	\$2,000	165.0	Semi	Central	Mor,Ten	Public
City 5	\$136,000	\$1,000	150.0	No	Fringe	Aft	None
City 6	\$142,000	\$1,150	157.5	No	Central	Mor,Ame	Public
City 7	\$250,000	\$2,000	195.0	Full	Inner	Bal,Ten,Ame	On-site
City 8	\$225,000	\$1,800	200.0	Semi	Inner	Mor,Aft	On-site
City 9	\$200,000	\$1,750	190.0	Semi	Fringe	Bal,Ten	On-site
City 10	\$146,000	\$1,300	187.5	No	Inner	Bal,Aft,Ten,Ame	Public

**Scores**

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1							
City 2							
City 3							
City 4							
City 5							
City 6							
City 7							
City 8							
City 9							
City 10							

**Answer: Jill**

A possible scoring system...

**Costs:**

- Membership - global scale (eg. 0 pts = \$0, 100 pts = \$100/year)
- Travel time - global scale (eg. 0 pts = 0 mins/wk,  
100 pts = 100 mins/wk)

**Benefits:**

- Sport Day(s) - local scale: Weekday = 0  
Weekend day = 50  
Both week and weekend day = 100
- Enjoyment - local scale: High = 100  
Moderate = 50  
Low = 0
- Physical fitness - local scale: High = 100  
Medium = 50  
Low = 0
- Organisation - local scale: Company team = 100  
5 work associates = 75  
3 work associates = 50  
1 work associates = 25  
None = 0
- Playing time - global scale: Time in minutes (per game)

**Answer: Apartments**

Table of scores:

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1	66	70	50	0	0	65	100
City 2	58	60	80	0	50	40	50
City 3	91	100	75	50	0	45	0
City 4	73	100	30	50	100	25	50
City 5	54	50	0	0	0	25	0
City 6	57	58	15	0	100	30	50
City 7	100	100	90	100	50	60	100
City 8	90	90	100	50	50	40	100
City 9	80	88	80	50	0	45	100
City 10	58	65	75	0	50	85	50

## *K Weighting*

For the overall score to be calculated, a set of weights must be placed on each parent criterion to represent the relative importance of each child criterion to the parent criterion's computed score. These weights represent a combination of the intrinsic worth, scoring scales and discrimination of each child criterion to the parent criterion. The weights represent the relative importance that each child plays, relative to its sibling criteria. The *swing weights method* described by Goodwin and Wright<sup>13</sup> provides the best procedure for determining weights. Val Belton<sup>14</sup> provides a further discussion of weighting the criteria.

When using local scales as the predominant scoring system, it is necessary to use the swing weights method. The reason for this becomes more apparent through providing an example:

Let's say you were thinking about buying a car and you have a budget of \$20,000 (a pre-emptive criterion). For simplicity, you have narrowed down your choices to 3 cars (alternatives):

- Car One has a capital cost of \$20,000 (V·I·S·A score = 0) and has a running cost of \$50 per week (V·I·S·A score = 100).
- Car Two has a capital cost of \$19,500 (V·I·S·A score = 100) and has a running cost of \$75 per week (V·I·S·A score = 50).
- Car Three has a capital cost of \$19,750 (V·I·S·A score = 50) and has a running cost of \$100 per week (V·I·S·A score = 0).

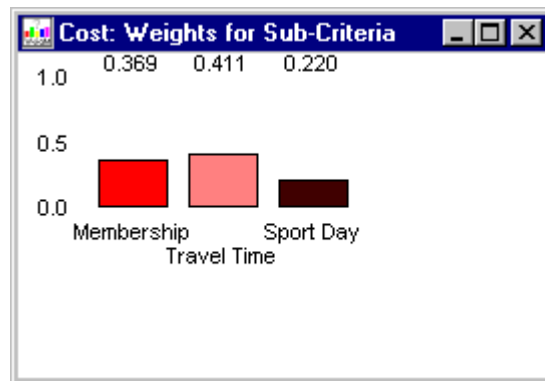
Both capital cost and running cost provide two distinct criteria for the selection of the most desired car. In weighting the two criteria it is necessary to look closely at the nature of the two criteria and what they are telling you about their relative importance. While the capital cost is important, it should be taken into account that all three cars meet your \$20,000 budget requirement. Secondly and most importantly, the range of capital costs represented by the 100-point VISA scale is small (\$500) in comparison to the range of running costs represented by the 100-point VISA scale (\$50 per week = \$2600 per annum). Therefore, while the capital costs is important, the **difference between the end points of the capital cost scale** is not as significant as the **difference between the extreme points on the running costs scale**, by a factor of around 5. Therefore, given the way we have scored the alternatives, it would be appropriate to assign the capital cost criterion with a much smaller weighting than the running cost criterion.



It is a good idea to view the Profiles chart before deciding the weights. What is important when setting weights is the difference between the two ends of the scoring scale (the best and worst alternatives if local scales have been used) on each of the criteria scales. This chart shows graphically the best and worst alternatives on the different criteria, and if the decision-maker is familiar with the differences in reality between these extremes, such a plot can be helpful when deciding weights.

V·I·S·A allows the decision maker(s) to input and assess weights in a number of ways. To enter the weight, open the dialogue box for the parent criterion whose children criteria weights you wish to enter or vary. Open the dialogue box by left clicking on the criterion. Select the *Weights* option, then change a weight by:

- Positioning the cursor at the top of the bar and dragging the weight to the desired position; or
- Click above bar and weight will move to that level: or
- Input numerical setting by selecting *Settings* menu and then *Numerical Value*. Click on the value of the weight to be set and enter desired value in the box that appears. The value of the weight will be fixed until changed.
- Be careful not to click anywhere in the weights bars area unless you intend to change the weights!



To fix a weight, Right-click on the criterion name or anywhere on the bar. The name will appear in red to show it is fixed. It can be freed anytime by right-clicking again.

To equalise weights, press SHIFT and right-click anywhere in a weight window. This will equalise all weights in that family, except the fixed weights which remain fixed.

The weights in each family will automatically be adjusted to sum to 1.

Save your model frequently, so that if you change your mind or make a mistake, you can restore a previous set of weights if you wish. The Snapshot feature in V·I·S·A will allow you to save different versions of

the weights, so you can try alternative sets of weights, perhaps representing different stakeholders, and switch between them.

When setting weights, do not let pre-emptive criteria influence the weights. Remember these criteria have already been met by all the alternatives in your model: any alternatives that didn't meet the pre-emptive criteria wouldn't still be under consideration now!

---

***Focusing Questions:***

- Is there some common yardstick for comparing one criterion with another? If not, are you prepared to trade-off one criterion with another?
  
  - If you had to make your decision on the basis of only one criterion, which criterion would you choose to use?
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  
  - Imagine an alternative that scores the worst on all criteria, and that you have been granted the chance of increasing one criterion to its best level. Which criterion would you most like to see increased from the worst to the best level?
-

- 
- The previous two questions should help you determine the most important criterion. Now delete that criterion from consideration for the moment, and repeat the above process. Can you use this process to rank the other criteria? Write down the criteria in each family in rank order.
  
  - To determine relative weights, you need to work out how important each criterion is relative to the most important criterion. How many points on each of the criteria (on their own scales) would you be happy to trade for say, 10 points of the top criterion? Use the swing weights method or Val Belton's guidelines to complete the process of assigning a set of weights for each family. Write down the relative weights.

- 
- Do the weights for each family sum to 1? If not, normalise them (see Goodwin and Wright).

- Do the weights refer only to the criteria as defined in your tree, free of influence from any pre-emptive criteria? If not, go back and adjust them.

***Example: Jill's back***

Use the swing weights method described by Goodwin and Wright to determine an appropriate set of weights for each of the sub-criteria in the 'Sport' model on page 42/43.

***Example: Apartments***

Use an appropriate method to determine the weights for each of the sub-criteria in your Apartment model.

**Answer: Jill's back**

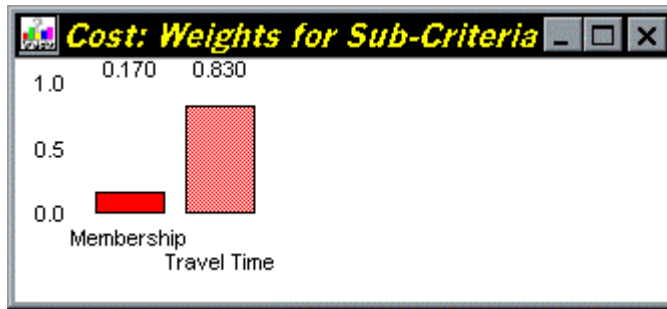
(Please note that this answer is not necessarily "the" right answer. It merely represents the authors' judgements.)

The first set of weights Jill decided to look at were the two sub-criteria under Cost. She had scored membership fees using \$100 = 100 points on the Membership scale, and 100 minutes = 100 points on the Travel Time scale, so the weights should represent her relative preference in trading off minutes for membership fees. In particular, equal weights would imply that she was indifferent between a \$10 decrease in Membership fee, and a 10 minute decrease in Travel Time. She quickly realised that the Membership fee is an annual cost, and she would rather save 10 minutes in travel time than save \$10 on Membership fees for the year. So this suggests a higher weight for Travel Time than for Membership fees. But she also thought that deciding on actual weights would be easier if she could measure both travel time and membership fees in weekly amounts. She suddenly realised that the weight would depend on which sport she was considering, because some sports involved traveling twice a week, and others once a week. In other words, her preferences between these two criteria were dependent on a third criterion (Sports Day(s))! This breaks the requirement that her model's criteria be "mutually preference independent".

In order to set the weights of these two criteria, she must change her model to make the preference between them independent of any other criterion. She did this by redefining the Travel Time to be Travel Time *per week*, and rescored Rowing and Soccer (to scores of 90 and 60 respectively). Now she could compare a weekly reduction in Travel time with a weekly reduction in Membership fee. She calculated that 50 points on the Membership scale represented roughly \$1 per week, and felt that a saving of this amount was equivalent to a saving of 10 minutes per week in Travel Time. This means the weight for Travel Time should be 5 times that of Membership.

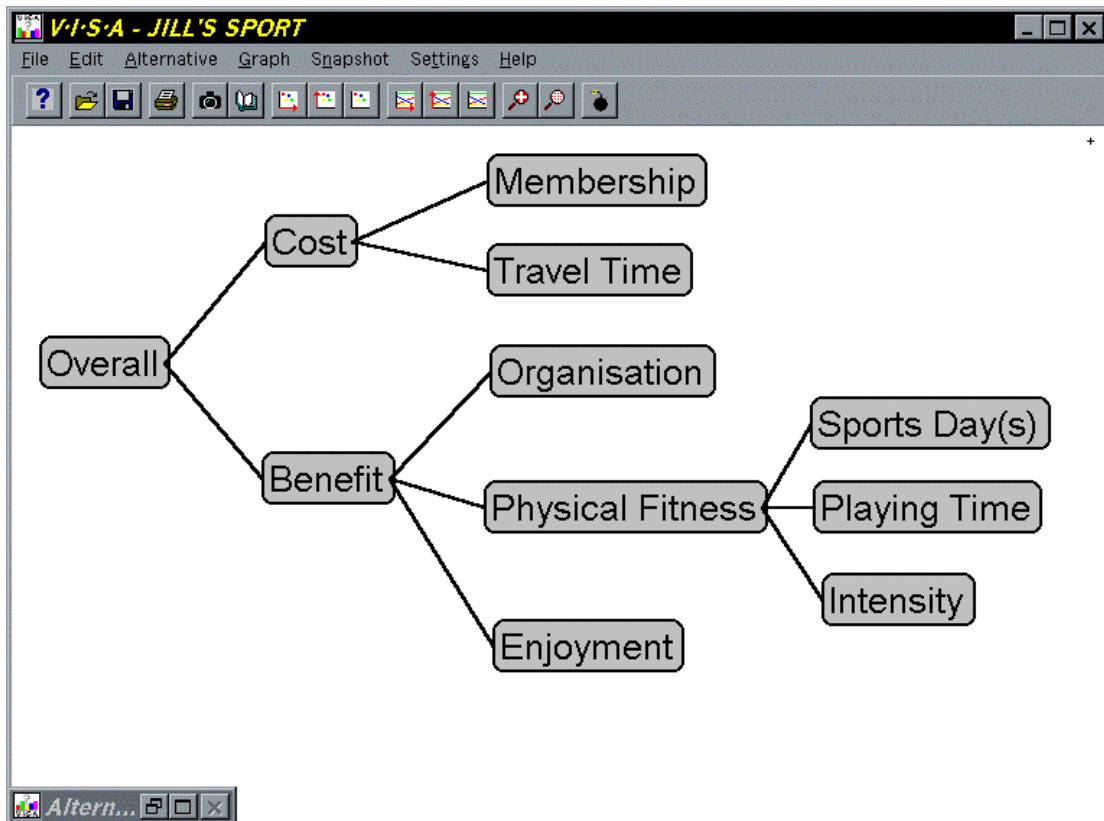
	Relative Importance	Weight
Travel Time	5	.83
Membership	1	.17
<i>Sum</i>	6	1.00

She entered the weights into VISA as shown below:



Next Jill considered the weightings for the child-criteria under the Benefit criterion according to their relative importance.

Again, looking at her model more closely, she realised that Physical Fitness was probably a result of playing time per game, the number of sessions per week, and a further factor, exertion required or intensity of the sport. She thought that while these may not be strictly additive (they might be multiplicative) she'd try a simple additive model to start off with. She therefore modified her tree as below:



She scored the criteria as shown in the alternatives window below:

	Membership	Travel Time	Organisation	Sports Day(s)	Playing Time	Intensity	Enjoyment
Netball	35	15	75	0	40	70	50
Rowing	50	90	50	100	60	60	0
Volleyball	15	15	25	50	60	20	0
Soccer	30	60	0	100	90	80	50
Indoor Netball	60	10	100	0	40	100	100

She noted that the convenience aspect of the Sports Day(s) criterion doesn't quite fit with the new interpretation of the criterion as a contributor to Physical Fitness, but she thought it shouldn't make much difference since it was at the lowest level of the tree. She could later take the convenience factor out of this criterion and add it to the Enjoyment criterion (For simplicity we'll leave it as it is at present).

In assessing the weights of the three sub-criteria of Physical Fitness, Jill looked at the ranges:

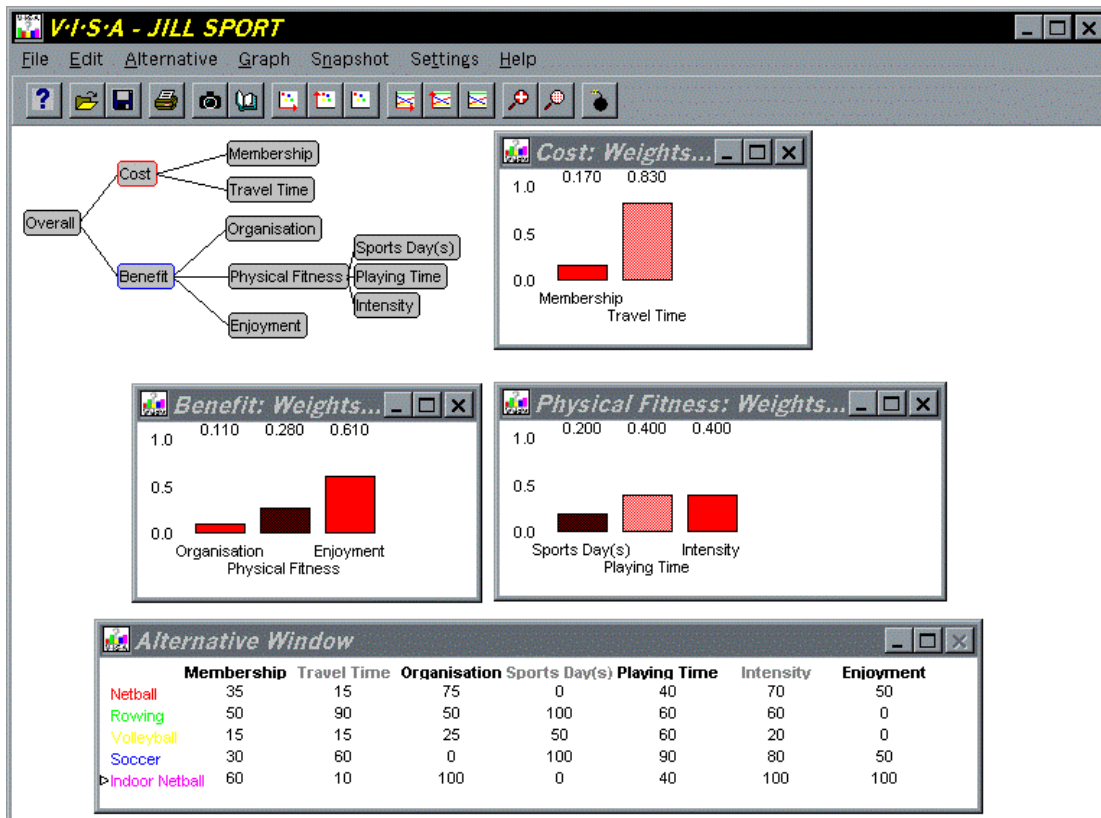
Sub-Criterion:	Range represented by the 100 point scale:
Sports Day(s)	1-2 days per week
Playing Time	0 - 100 minutes per game
Intensity	up to 100 for Indoor Netball

She gave Sports Day(s) half the weight of the other two (as it uses a local scale which exaggerates the differences between the alternatives' scores), arriving at weights of .2, .4 and .4 respectively.

Finally Jill felt that she valued Enjoyment ahead of the other criteria and so gave it the highest weighting; even though her main reason for taking up a sport is to improve her physical fitness, she felt that the difference between the fitness obtained from the 5 sports was not as important to her as the difference between a high enjoyment and low enjoyment sport. Next she determined that the Physical Fitness criterion was the next most important to her and so gave it the second highest weighting. And lastly came Organisation. She wasn't sure of actual weights so thought she'd try, as a starting point, the Rank Order Centroid method of Edwards and Barron<sup>15</sup>, assigning weights of .61, .28 and .11.



Her full model is shown below.



**Answer: Apartments**

(Please note that this answer is not necessarily the right answer. It represents the authors' preferences)

The four graduates, in deciding which weights to assign each of the criteria in their V·I·S·A model, began with the criteria under the Cost criterion. They noted that while the Purchase price of an apartment is a one-off cost, the associated Body Corporate fee is an annual fee. These costs need to be put on the same footing, eg annual costs, and one way of doing that is to replace Purchase prices by annual mortgage costs (strictly one should add insurance costs etc, but let us assume that they will be the same for all apartments, ie a non-discriminating factor). Next, they examined the ranges represented by the scoring scales for Purchase price and Body Corporate fee. The Purchase price scale represents a range of \$250,000 in purchase prices. At current interest rates, this would equate roughly to a range of \$20,000 pa in mortgage costs. The BC

fee scale represents a range \$2,000 pa. Hence a change of 5 points on the Purchase price scale is roughly equivalent to a change of 50 points on the BC scale. This provides a starting point for the relative weights. If there are other non-financial factors influencing the weights, they could then be incorporated, eg they may feel happier parting with \$1 towards a mortgage than \$1 towards the BC fee, in which case the weighting may be a bit less than 10:1. In the absence of any such indications, let's just use the relative values calculated above as the weights.

	Relative importance	Weight
Purchase \$ Annual Cost	100	0.91
Body Corp. fee	10	0.09
<i>Sum</i>	110	1.00

Next, the four graduates determined the weighting for each of the criteria under the Benefits criterion. The graduates used the swing weights method. First they imagined an alternative that scored the worst on all the Benefits criteria. It was the smallest, was unfurnished, was on the Fringe, had the fewest added features, and had no parking. If they were allowed to improve one of these criteria to its best value, which would it be? They chose parking, firstly because of the intrinsic worth of having convenient parking, as they would be using the Apartment when visiting Wellington from outside the city, and secondly, the difference between the parking facilities offered by the apartments was considered very significant. Furnishing was deemed to be the next most important criterion, followed by Location (although Location is usually intrinsically important, the graduates felt there was little real difference between the Apartments on offer in terms of Location). The Size and Added Features of the Apartment were judged to be of last equal importance. Using the swing weights method, the graduates agreed upon the following set of weights:

	Relative Importance	Weight
Parking	100	0.38
Furnishing	80	0.31
Location	40	0.15
Size	20	0.08
Added Features	20	0.08
<i>Sum</i>	260	1.00

## Initial Results

### *Objectives:*

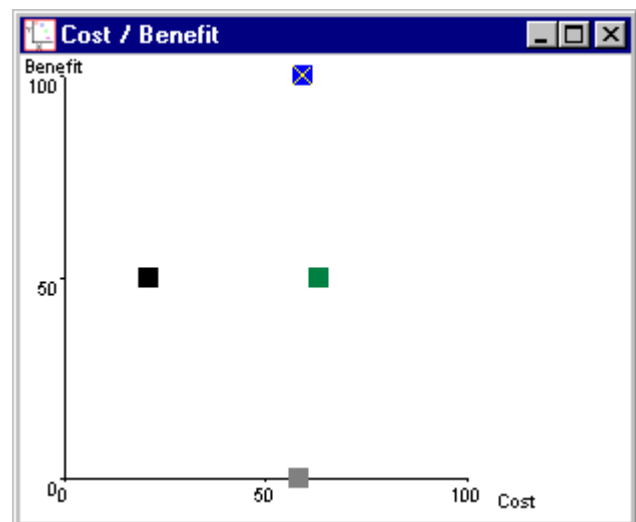
- Students will be able to display their initial results in a fashion that is appropriate for the type of decision problem.

K The simplest means to display the results and determine which alternative is best is through the Thermometer and Bar Chart option. By clicking on the Overall Score criterion the decision maker(s) can select either the Thermometer or Bar Chart option to display the results.

However, the Thermometer and Bar Chart option may not provide the correct results that the decision maker(s) is looking for. This is especially the case when the decision maker(s) are concerned with relative efficiencies in an Input/Output model or Cost/Benefit model setting. In this case it is more appropriate to produce an X-Y plot, to represent the relative efficiencies of the alternatives. (The Overall score will be an indicator of the largeness of the alternative, not its relative efficiency.) X-Y plots are also useful for checking to see the trade-offs between any pair of criteria.

K To create an X-Y plot:

- Click on the X-axis icon in the toolbar (next to the book icon) and move the cursor to the criterion you want to select for the X-axis, and click. Click on the Y-axis icon, and select the criterion for the Y-axis. To display the graph, select Create X-Y Performance Graph (third icon in grouping).



- Note that the X-symbol, found within one of the alternative plots of the X-Y plot, indicates the presently selected alternative in the Alternative Window. Also note that if you save the V·I·S·A model after creating an X-Y plot, the plot will not be saved!

From this graph it is possible to gain a better understanding of the situation at hand. The X-Y plot will graph each alternative depending on the X and Y-axis selections the decision maker(s) have made. Each alternative can be selected to find out what it is by clicking on the coloured square. The relative cost-effectiveness of an alternative is shown by its place on the X-Y plot. An alternative with high costs and low benefit is less effective at converting resources into outputs than an alternative with low costs and high benefits.

In many cases it may be found that the alternative with the highest overall score may not be the best solution, because even though it's the largest alternative, it may not be the most cost-effective. Instead an alternative that does not have a high Overall score, may be a better option as the Benefits or Outputs gained from it may be proportionately higher than the Costs or Inputs associated with it.

The X-Y plot also helps the decision maker(s) identify alternatives that are being dominated, in any model. An alternative is dominated when another alternative scores the same or better than it on every criterion. Any dominated alternatives should be removed from further analysis, therefore reducing the set of alternatives and further focusing the decision problem.

There will usually be a small number of alternatives that perform relatively well, and lie on what is referred to as the "efficient frontier", eg see Goodwin and Wright<sup>16</sup>. (Note this use of "efficiency" has a more precise definition than the lay meaning used above). Decision-makers would be wise, generally speaking, to choose between these alternatives on the efficiency frontier. However, Val Belton does discuss the case of the "linearity trap", where an alternative can just miss being on the efficiency frontier, but actually represent a better balanced decision than those on the frontier, because the latter are more extreme.

Strictly scores on all criteria at all levels should be considered before declaring an alternative "dominated", but for the purposes of the following exercises, the notion of efficiency and dominated alternatives will be relaxed to apply just to the top level criteria.

---

***Focusing Questions:***

- What type of answer does the decision maker(s) require? Is it the best overall score, the best all rounder, or the best performing option (i.e. Ratio of Costs to Benefits/Inputs to Outputs)?
  
- Which graph best suits the required answer?
  
- Which is the best alternative to choose? Why?

- 
- If an X-Y plot has been drawn, are any alternatives dominated? If so, which are they?

- Which alternatives lie on the efficient frontier?

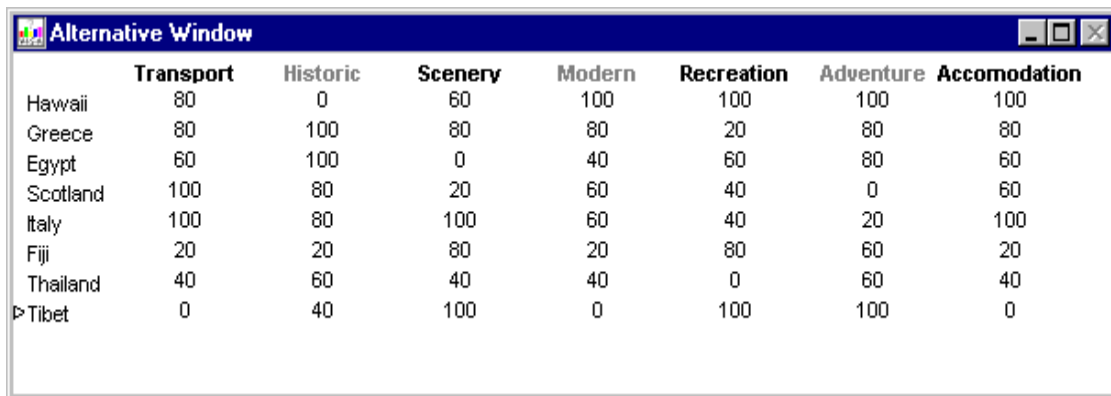
### Example: Brendan's holiday

Brendan was in an excited mood. He was lounging in his swimming pool recliner with a laptop on his lap and an ice-cold beer in one hand. Being of the intelligent type, Brendan had opened his V·I·S·A package on his laptop and had begun developing a model suited for his holiday dilemma.

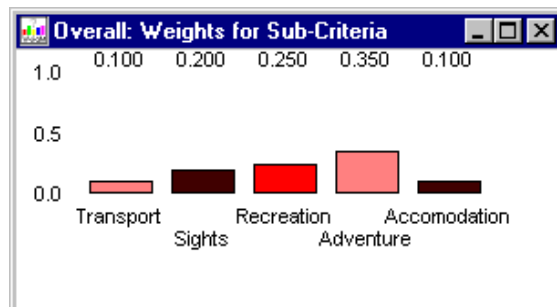
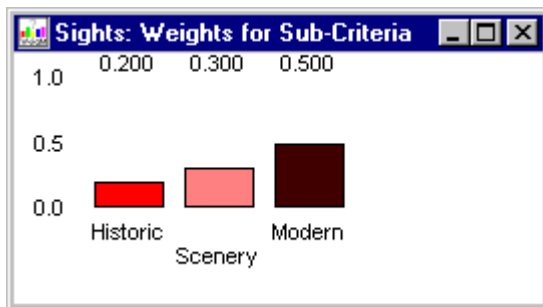
After a couple of hours tapping away, interspersed with breaks to reapply sunscreen and restock his beer cooler, Brendan had developed a criteria model that he thought was particularly befitting of his situation. He had scored each alternative against each of the criteria and assessed weights that he thought were representative. Yet, Brendan was unsure how to determine which alternative was best and so asked his girlfriend. By doing so he was now locked into a situation where she would have to go with him.

However, his girlfriend didn't know either, but suggested he try ringing some of his friends who were more knowledgeable in the field of decision making. Reaching for his mobile phone, Brendan began ringing a few of his friends to find out the best method to use.

Develop the 'Holiday' criterion tree within V·I·S·A that is presented on page 26. Input the scores and weights as shown.



	Transport	Historic	Scenery	Modern	Recreation	Adventure	Accommodation
Hawaii	80	0	60	100	100	100	100
Greece	80	100	80	80	20	80	80
Egypt	60	100	0	40	60	80	60
Scotland	100	80	20	60	40	0	60
Italy	100	80	100	60	40	20	100
Fiji	20	20	80	20	80	60	20
Thailand	40	60	40	40	0	60	40
Tibet	0	40	100	0	100	100	0



Use the focusing questions to select the best alternative to make.

- What type of answer does the decision maker(s) require? Is it the best overall score or the best performing option (i.e. Ratio of Costs to Benefits/Inputs to Outputs)?
- Which graph best suits the required answer?
- Which is the best alternative to choose? Why?
- If an X-Y plot has been drawn, are any alternatives dominated at this level? If so, which are they?
- Which alternatives lie on the efficient frontier?

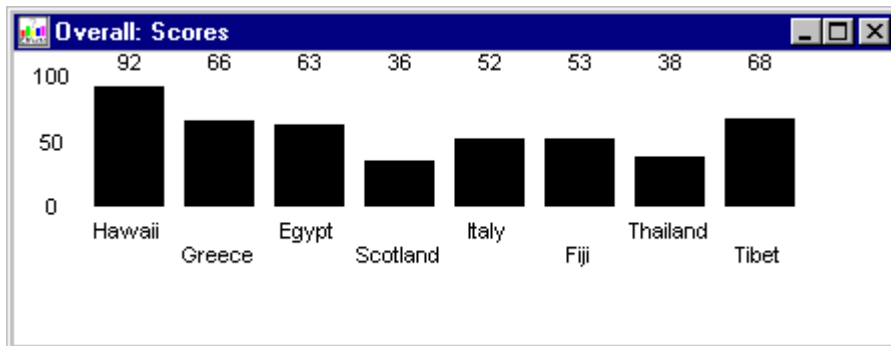
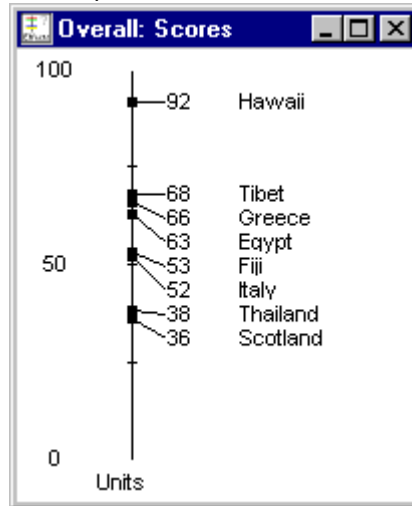


***Example: Apartments***

- Using the Apartment model, which alternative has the highest overall score? What does it mean to have the highest overall score in this model? (Hint: how were costs scored? Does it make sense to look at the Overall scores? If not, what would be more appropriate?)
  
- Which alternative is best in a Cost/Benefit plot? Why?
  
- Which alternative is the better option to take?
  
- In the Cost/Benefit plot, which alternatives are being dominated?
  
- Which alternatives lie on the "efficient frontier"?

**Answer: Brendan's holiday**

The best display tool to use would either be the thermometer or bar chart. An X-Y plot is not a useful display tool, as the model isn't broken up into Costs and Benefits. Through viewing the thermometer gauge, the best alternative is Hawaii with an overall score of 92. Second best is Tibet on 68 and Greece (66) ranks third. As the gap between Hawaii and Tibet is significantly large, no further analysis appears necessary. Hawaii is therefore the best holiday for Brendan to take.



***Answer: Apartments***

*{These may change depending on the set of weights devised above for the answer}*

In this case, it doesn't make sense to use the Overall scores, as all this will favour the apartments with the highest benefits and the highest costs. The former is desirable, but the highest cost is undesirable. So for this sort of model where Cost has been scored this way around, the Overall scores should be disregarded and the X-Y plot of Cost-Benefit will give the best measure.

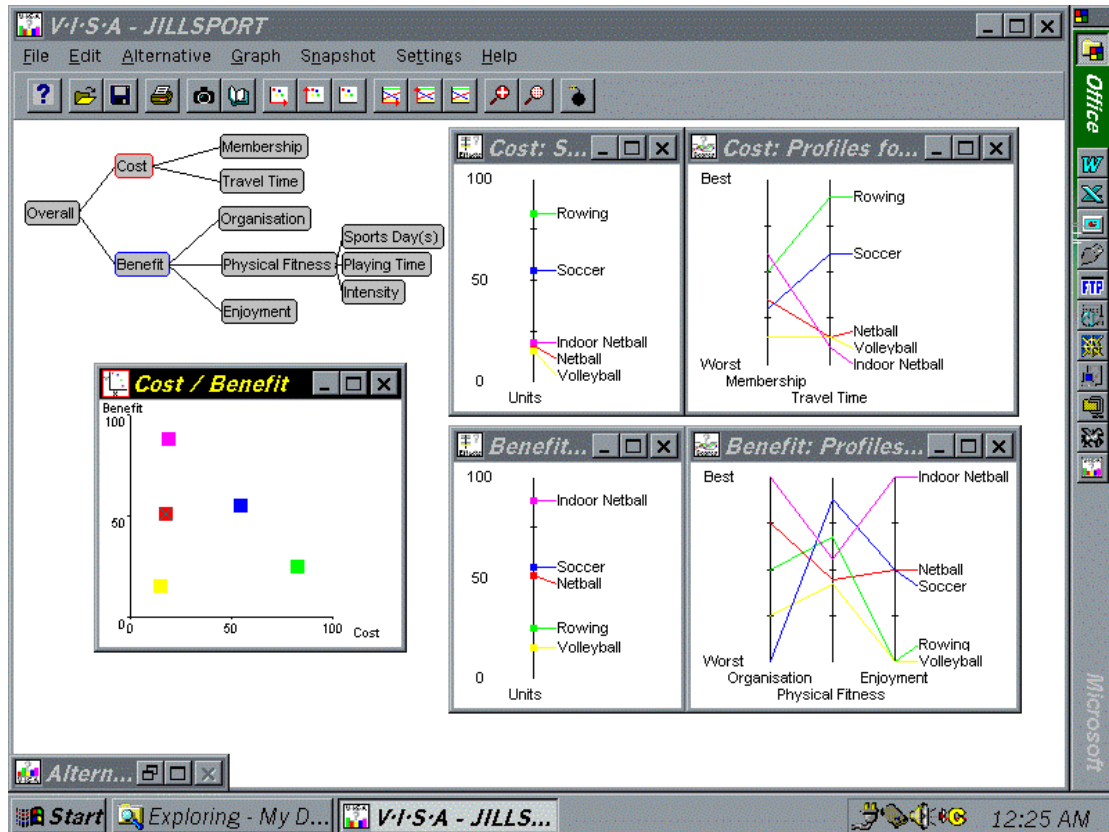
Specifically:

- City 7 has the highest Overall score. But this is meaningless, as although City 7 has the highest Benefits, it also costs the most.
- Hence the Cost-Benefit plot will be more useful, given that we've set the scores for cost with a high cost getting a high VISA score, and a low cost getting a low VISA score.
- From the Cost-Benefit Plot, and with their original set of weights, City 7 actually has the highest Benefit-Cost ratio.
- The alternatives on the efficient frontier at this level are City 5, 6, and 7, but few people would be expected to choose City 5 over City 6, since City 6 has far greater benefit for very small additional cost.
- City 3 is clearly dominated by several apartments that score better in terms of both Benefits and Costs.
- City 2, 6 and 10 almost coincide on the Cost-Benefit plot. They all provide similar value for money. City 2 and 10 are just dominated by City 6, but are so close to it, they are effectively the same.
- City 1, 4, 9 and 8 lie just below the Efficient Frontier. Belton (1990, p74) warns us to beware of discarding them prematurely: while they appear to not be on the Efficiency Frontier, they may still be good all-round alternatives. They may fall into what Zeleny calls the Linearity Trap (see Belton, p74).
- Apart from City 3 and City 5, most of the apartments give similar benefit for each dollar spent - and the deciding factor may well be how much the graduates want to pay, or some other criterion outside the model!

*{These answers may change depending on the set of weights used for the model}.*

**Answer: Jill**

Jill's model gave the following results:



The screen shows the Cost and Benefit thermometers, as well as a Cost/Benefit chart. This would indicate that probably Indoor Netball is the best option as it scores the best on Benefits, and has pretty low costs. Netball and Volleyball have slightly lower costs, but they have much lower benefits. (She's also shown the Profiles charts for Costs and Benefits which we'll come to in the next section).

## Sensitivity Analysis

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### *Objectives:*

- Students will be able to perform a detailed sensitivity analysis using the various features available within V·I·S·A.
- 

The purpose of sensitivity analysis is to check how robust the preferred option is to changes or errors in the data, especially the weights. In other words, what's the chance of making the wrong decision, and how wrong can the decision be? It also provides a chance for the decision-maker to more fully understand the decision or system being modelled and the important characteristics influencing the decision. In a moment, we'll tell you how to perform the sensitivity analysis within V·I·S·A, but first let's give an overview of what we're trying to achieve.

First up, it's important to check that the **model** is working the way you'd expect it to. As you give more weight to a criterion, the options that scored well on that criterion should do better. Just check that this happens, and that the changes seem reasonable.

Secondly, it is very useful to check whether the overall **results** seem reasonable in hindsight. It's often easier to justify the results after the event, even if the decision was too complex for a judgement beforehand. If the result does not seem reasonable, then you need to check whether you've missed any criteria, or whether the tree structure may be inappropriate, or the scoring or weights are way out. In a moment we'll show you how you can check the effects of getting the weights wrong.

Another thing to look for is whether the favoured option is a clear winner, or whether there are other close options. Sometimes the options seem to fall into groups: maybe leaders, laggards, and middle bunch. This can be quite useful for learning about the problem: there may be general characteristics in common within the groups.

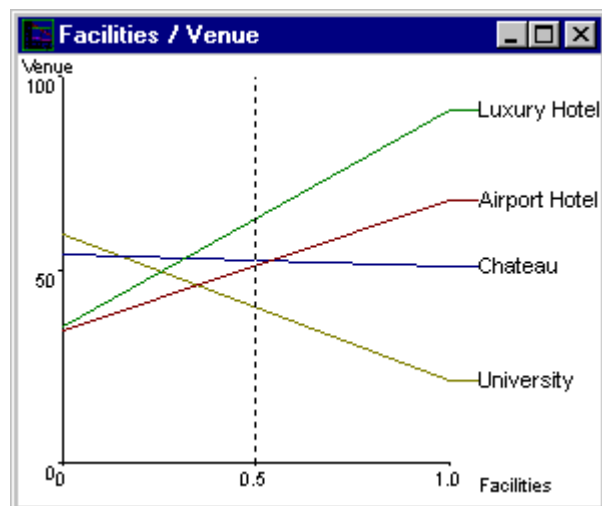
Now for the How-to! The V•I•S•A package allows the decision maker(s) to perform a sensitivity analysis on the results to investigate these issues. Of particular interest is the effect of errors or changes in the weights placed on the criteria. The scales used for scoring on the end-criteria should also be looked into further.

K To determine how robust the results are to the weightings, the decision maker(s) can perform a visual check. This is achieved by:

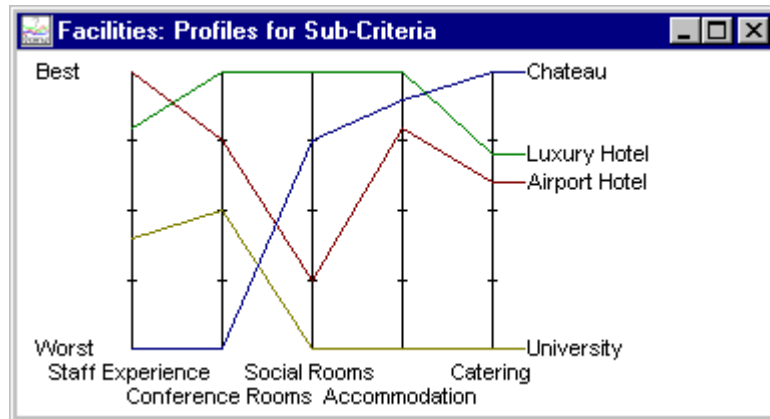
- Leaving the X-Y plot, Thermometer or Bar Chart on the screen.
- Selecting any of the parent criteria and selecting the *Weights* option. Then interactively altering the weights in the Weights window through to their extremes and checking the results screen for any changes in the best option.

K In addition, V•I•S•A supports several other Sensitivity Analysis features. These are:

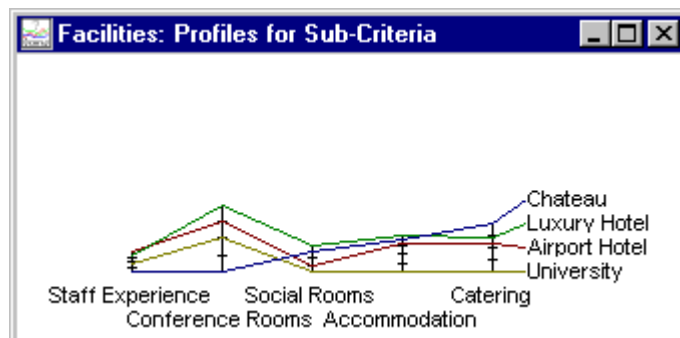
- Sensitivity Graph: shows the effect of changes in the selected weights on scores aggregated to the selected level.<sup>17</sup> To plot a sensitivity graph click on the *Choose Weight for Weight Sensitivity Graph* icon in the toolbar. Move the cursor to the sub-criterion (child) whose weight is to be varied, and click. Then click on the *Choose Score for Weight Sensitivity Graph* icon in the toolbar. Move the cursor to the criterion whose score is to be observed, and click. Note that this criterion must be either a parent, grandparent and so on, of the sub-criterion selected above. (If you get horizontal lines in your graph, check whether you observed this requirement!) Click on the *Create Weight Sensitivity Graph* icon to view graph. The vertical dotted line represents the current weight setting for the sub-criterion under the parent criterion.



- Profiles: shows the profile of scores for each alternative on all sub-criteria in the selected family.<sup>18</sup> To create a Profile of the sub-criteria, click on any parent criterion and select *Profiles* in the dialog box.



- Weighted Profiles: the heights of the vertical lines corresponding to the criteria are scaled in accordance with the criteria weights. In a similar fashion to the Profiles plot, click on any parent criteria and select *Weighted Profiles* in the dialog box.



- Data Envelopment Analysis (DEA): DEA is another technique that is sometimes used to measure the relative efficiency of a number of similar units (alternatives) performing essentially the same task.<sup>19</sup> There is an add-in DEA feature in V·I·S·A which is a useful tool for Sensitivity Analysis in an Input/Output or Cost/Benefit model. It determines whether an alternative can be moved to the efficient frontier under any weight setting. To use the DEA feature, complete an X-Y plot as described above. Save your model, as the DEA option will change your weights. By right-clicking on any alternative, V·I·S·A's DEA feature will determine if the alternative can be moved to the efficient frontier.

If this feature doesn't work, it is probably because the feature hasn't been installed on your machine - contact your instructor or the software suppliers for instructions on how to add it in.

- K Note that when saving your V·I·S·A model these sensitivity analysis charts will not be saved. Also note that several sensitivity analysis charts do not always change after you have changed the weights. It is necessary to check each graph after you have changed the weights and make sure that the graph has subsequently changed also. If it appears that the graphs haven't changed then you will have to redo them to get the necessary set of results.

---

***Focusing Questions:***

- Does the model behave as you'd expect it to as you change the criteria weightings?
  
  
  
  
  
  
  
  
  
  
- Are there some weights you are particularly unsure about? Where does this uncertainty lie?



- 
- Are the linear and/or non-linear scales used suitable? Could they be altered or changed?

- Does a visual check of the weights reveal any changes in the order of the overall scores for each of the alternatives? If so, what are they?

- From the sensitivity graphs, over what ranges of weights does the best overall alternative remain the best? In what circumstances would it not be best?

- 
- Overall, do the results seem reasonable intuitively? If not, why not? Do you need to add a criterion or two? Or change the tree structure? Or revisit the scoring scales?
  
  - If it does seem reasonable, does it enhance your understanding of your choices and preferences?
  
  - Are the results clearcut or close? Is the preferred option a clear winner, or is there a close second? How close is the next best option?
  
  - Are there apparent groups of options? Are there any general characteristics in common in these groups?
-

- 
- Do the profiles and weighted profiles suggest any alternatives are dominated by other alternatives? What are they?

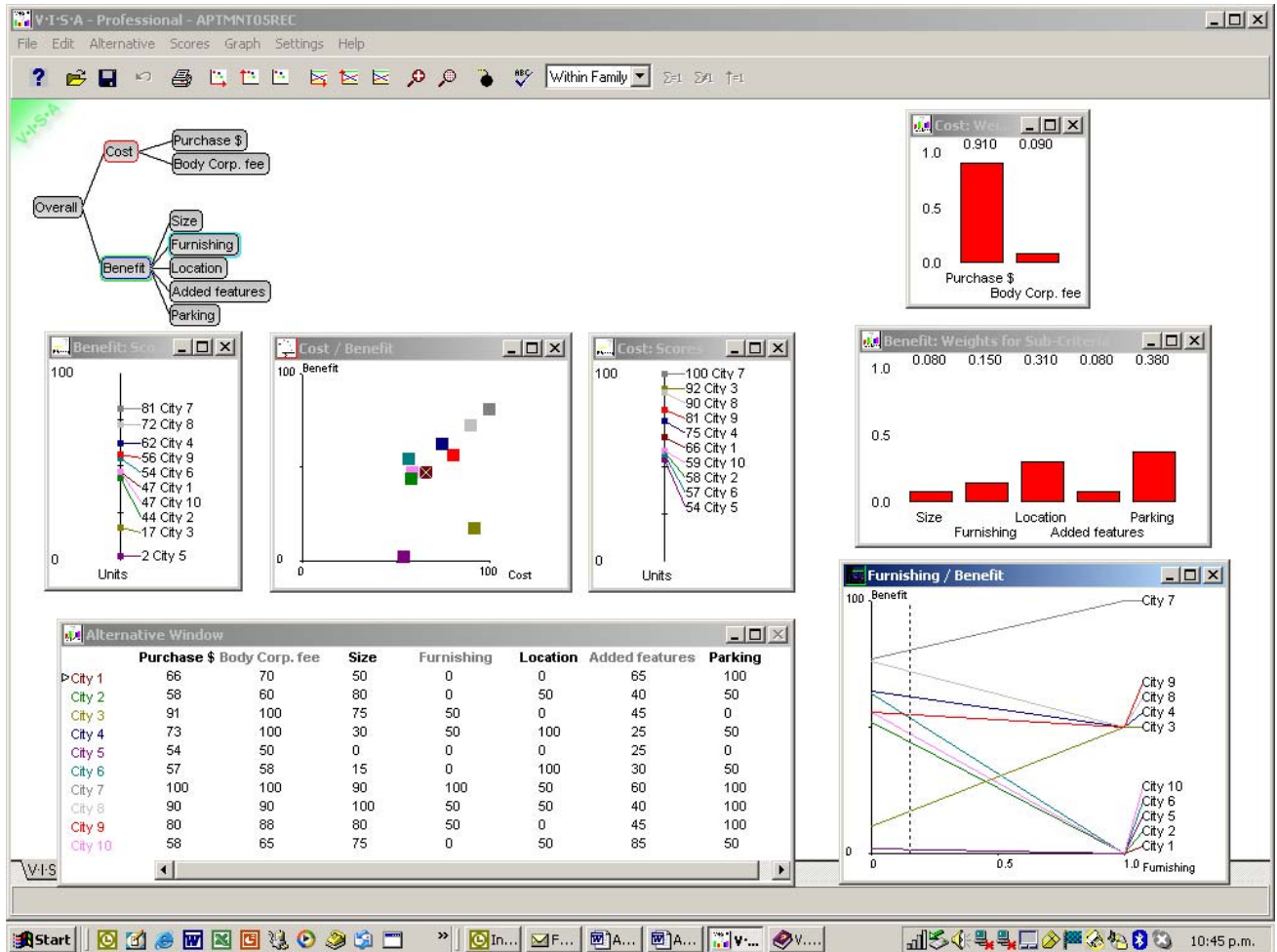
- If relevant, does the DEA feature suggest that a dominated alternative can be moved to the efficient frontier? If so, are the weight settings reasonable, or too extreme? Should further analysis be carried out to determine the significance of this finding?

### ***Example: Apartments***

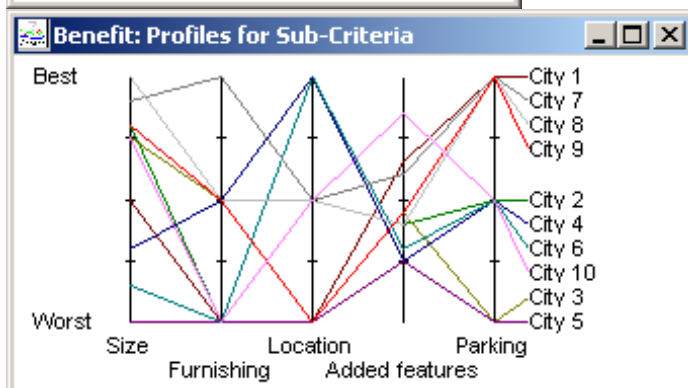
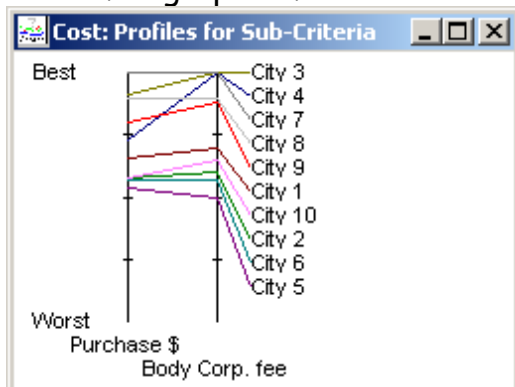
Having determined through an X-Y plot that Apartments 2, 6 and 10 seem the best alternatives to select, given that they would rather opt for the lower priced apartments if reasonable, the graduates remained uncertain as to how accurate their weighting system was, and how robust these options would be to changes in the weights. To check the impacts of this uncertainty, the graduates decided to perform a Sensitivity Analysis.

The graduates decided to plot the profiles charts for Costs and Benefits, and Sensitivity graphs of Purchase Price weight Vs. Cost score and Furnishing weight Vs. Benefit score.

This showed that if the weight given to Furnishing was reduced, then City 6 would perform better on Benefits, and become the most attractive option in terms of relative efficiency (best Cost-Benefit ratio). The Efficient Frontier for the set of weights shown in the screenshot below comprises City 5, City 6 and City 7, as before. City 6 would again most likely be preferred to City 5 because for a very small increase in Cost, the Benefits increase dramatically. If the graduates can afford City 7, it does have higher benefits, but the graduates need to ask themselves whether the additional benefits gained by choosing City 7 over City 6 justify the additional cost. Given this new set of weights, the graduate's preference for City 6 over City 7 now seems reasonable in that City 6 has a higher Benefit-Cost ratio.



- Profile graphs of the Cost and Benefit criteria.



Complete the above graphs and perform the DEA analysis. Apply the focusing questions and determine if the graduates' weighting system is robust.

- Does the model behave as you'd expect it to as you change the criteria weightings?
- Are there some weights you are particularly unsure about? Where does this uncertainty lie?
- Are the linear and/or non-linear scales used suitable? Could they be altered or changed?
- Does a visual check of the weights reveal any changes in the order of the overall scores for each of the alternatives? If so, what are they?
- From the sensitivity graphs, over what ranges of weights does the best overall alternative remain the best? Beyond this, which alternative becomes best?


- Overall, do the results seem reasonable intuitively? If not, why not? Do you need to add a criterion or two? Or change the tree structure? Or revisit the scoring scales?
- If it does seem reasonable, does it enhance your understanding of your choices and preferences?
- Are the results clear-cut or close? Is the preferred option a clear winner, or is there a close second? How close is the next best option?
- Are there apparent groups of options? Are there any general characteristics in common in these groups?
- Do the profiles and weighted profiles suggest any alternatives are dominated by other alternatives? What are they?
- If relevant, does the DEA feature suggest that a dominated alternative can be moved to the efficient frontier? If so, are the weight settings reasonable, or too extreme? Should further analysis be carried out to determine the significance of this finding?

## Quick Guide to V·I·S·A

You can download a limited evaluation version of V·I·S·A off the web at <http://www.simul8.com/products/visa.htm>

This will expire after 1 month. The full version is also available.

### *1. Getting Started*

- ➡ Open the V·I·S·A program.
- ➡ To open an existing V·I·S·A model, choose either the  icon on the toolbar, or use the OPEN option within the FILE pull-down menu.
- ➡ To create a new V·I·S·A model, CLICK on the RIGHT mouse button. This will create an Overall criterion.

### *2. Creating a criteria hierarchy*



- To add a new criterion, CLICK on the RIGHT mouse button anywhere in the main window.
- To (re)name or alter the properties of the criterion, CLICK on the LEFT mouse button while the mouse pointer is on the criterion.
- To move the criterion, simply CLICK and HOLD on the LEFT mouse button and then DRAG the criterion to the desired position.
- To delete a criterion, click on the BOMB icon on the toolbar, and place the bomb icon on the criterion you wish to delete. Beware, any subcriteria attached to that criterion (ie. its children) will be deleted too.
- To change a parent of a criterion, CLICK and HOLD on the RIGHT mouse button on the criterion then DRAG the cursor to the new parent.
- Criteria are loaded into the Alternatives (scores) window as they are created. To change this order, use the MOVE buttons in the criterion's dialog box.
- When you have finished creating your criteria tree, use the TIDY option within the EDIT pull-down menu to tidy your tree.
- To enlarge or reduce the size of your tree, select the appropriate MAGNIFYING GLASS icon in the toolbar.

### ***3. Entering alternatives***

- To add Alternatives, select the Alternatives window and increase its size to an appropriate size for you to enter the Alternatives.
- Alternatives can be added by either:
  1. Selecting the ADD option within the ALTERNATIVE pull-down menu.
  2. Selecting CTRL+A on your keyboard (shortcut key).
- To (re)name the alternative, select the alternative by CLICKING on the LEFT mouse button and change it to the desired name in the Dialog box.
- Through clicking on the COLOR icon within the Alternative dialog box, you can change the colour of the alternative.

## 4. Entering scores

Scores can be entered in several ways:

- Open the chosen criterion's dialog box and select either the THERMOMETER or the BAR-CHART (in pictorial form). Select the SHOW NUMERIC VALUES feature within the SETTINGS pull-down menu. Position the mouse pointer on an Alternative name and drag it to the desired position; or
- In the Alternatives window, enter the scores manually as you would for any spreadsheet; or
- Import data from spreadsheet, using IMPORT in the FILE pull-down menu. Note: V·I·S·A requires data to be saved as Text tab delimited .txt (or .csv) format, with a blank cell in A1, Criteria headings in Row 1, Alternative Names in Col A, and data in Row 2, Col B onwards. Save as a .txt file and close the Excel file before accessing from V·I·S·A.

## 5. Entering weights

Open the Dialog box for criterion whose subcriteria weights you wish to enter or vary. Display the weights bar chart by selecting WEIGHTS.

To change a weight:

- Select the SHOW NUMERIC VALUES feature within the SETTINGS pull-down menu (if you haven't already done so earlier).
- Then, either:
  - Position the mouse pointer at the top of each bar and drag the weight to its desired relative position; or
  - Click above each bar and its weight will move to that level; or
  - Manually enter the desired value in the number box that appears at the top of the chart. The value of the weight will be FIXED until changed.
- To Fix a weight, CLICK on the RIGHT mouse button on the criterion name or anywhere on the bar. The name will appear in red to show it is fixed. It can be freed anytime by CLICKING on the RIGHT mouse button again.
- To equalise all the weights in one family, simultaneously select the SHIFT key CLICK on the RIGHT mouse button anywhere in a weights window. This applies to all weights that are not already fixed (these remain fixed).

## ***6. Creating plots***

### Overall Scores:

- ➡ This is the main chart of results: click on the Overall criterion (or whatever the top level criterion is) and choose the BAR CHART or TEMPERATURE GAUGE option within the dialog box.

### X-Y Plots:

- ➡ Click on the CHOOSE X AXIS CRITERION FOR PERFORMANCE GRAPH icon on the toolbar and move the mouse pointer to the criterion you want to select for the and CLICK on the LEFT mouse button.
- ➡ Click on the CHOOSE Y AXIS CRITERION FOR PERFORMANCE GRAPH icon, and select the criterion for the Y-axis.
- ➡ To display the graph, SELECT CREATE XY PERFORMANCE GRAPH (third icon in the grouping).

### Profiles Graph:


- ➡ Open a criterion (must be a parent) dialogue box by clicking on the criterion, and select the PROFILES button.
- ➡ This will display scores of alternatives for all the criterion's subcriteria in one graph. The Profile Graph helps to see the strengths and weaknesses of individual Alternatives, and identify good all-rounders, and identify poor Alternatives that are dominated, or nearly dominated, by the better Alternatives.

### Sensitivity Graph:

- ➡ Click on CHOOSE WEIGHT FOR SENSITIVITY GRAPH icon in the toolbar, move cursor to the sub-criterion whose weights are to be varied, and CLICK on the LEFT mouse button.
- ➡ Click on CHOOSE SCORE FOR SENSITIVITY GRAPH icon in the toolbar, move cursor to the criterion whose score is to be observed, and click. This criterion must be a parent (or grandparent or great grandparent...!) of the sub-criterion whose weights are being varied.
- ➡ Click on CREATE SENSITIVITY GRAPH.

## ***7. Saving your Model***

- ➡ Use the SAVE option or SAVE AS option within the FILE pull-down menu.

- Select the  icon on the toolbar.
- To save the current layout of windows, select SAVE LAYOUT AS within the DISPLAY submenu within the FILE pull-down menu.
- To restore the saved display layout format, select RESTORE LAYOUT within the DISPLAY submenu within the FILE pull-down menu. Note that this will not restore your X-Y plots or sensitivity charts.

## ***8. Printing***

- Use the PRINT option within the FILE pull-down menu to print the hierarchy tree; or
- Select the PRINT icon on the toolbar to print the active window; or
- Use the PRINTSCREEN key on your keyboard (top right on most keyboards) to copy the whole screen picture to the clipboard, which can then be pasted into MS Word or MS Paint. (Note a screen dump of the V·I·S·A window is a BIG file: don't expect to fit more than 2 or 3 V·I·S·A pictures onto a 1.44 MB floppy disk); or
- To print a single highlighted window, simultaneously select ALT + PRINT SCREEN, and paste into MS Word as above.

## Apartments Module

The following module provides you with a complete guide to an integrated, fully operational V·I·S·A example. This will enable you to gain a full understanding of V·I·S·A and its features. The module is the same example as provided throughout this Workbook - just brought together in a simple and complete form for your own benefit. We advise that you use this opportunity to learn first hand how V·I·S·A can aid you in your own decision problem.

The example that will be used is that of choosing between apartments in the Wellington inner city area. The example was initially proposed by four students in the Victoria University MBA Problem Solving and Decision Analysis course in 1997. We have developed it further for use here. The problem identification process undertaken by the students was built on the following scenario:

After completing the Victoria University MBA program, a group of four graduates secured good jobs with organisations that are based in Wellington. While discussing their jobs with each other they found they were all facing the same concerns over living arrangements. All four were able to carry out a large portion of their work at home, and this has led to all of them purchasing 'lifestyle' properties based in various locations around the lower North Island. When required in Wellington for meetings and the like, each graduate would travel down and spend a few days in Wellington living out of a hotel room. The cost of this accommodation fell on the graduates, as it was their decision to live out of the Wellington area.

Following lengthy discussion and research, the four graduates found that if they combined resources they could purchase an inner-city apartment for approximately the same cost as the expenses of a hotel room. Purchasing an apartment would enable the graduates to live in the area when they wanted - mainly when work commitments required. Furthermore, they found that their work commitments differed during the year for each person. Finally, the apartment option was seen as an advantage as it would build up equity in an asset at the same time.

Agreement was reached to look for an apartment that would present the best overall value for money and ease of use for all group members. The

problem remained how to choose an apartment that would suit everyone's preferences...

Does the above scenario suggest MCDA would be appropriate? Apply the following focusing questions to decide.

- What is the significance of the problem or dilemma? Is it a small problem that is suited to an intuitive approach, or does it warrant a more formal approach?
- Can the problem or dilemma be broken down into a number of key components (criteria)? What are they? Do some of the criteria appear to be in conflict with each other?
- Does the problem or dilemma involve several alternatives that need to be compared?
- Are these alternatives already identified, and if not, could they be identified fairly readily?
- What is your pre-conceived judgement of the problem?
- Would the MCDA approach and V·I·S·A decision support tool be useful in providing further insight for the final decision?

***Answer***

Yes it would. MCDA would provide a sensible and logical framework for determining the best overall apartment. A range of alternatives (possible apartments) and a set of criteria could be developed from a variety of resources ("hard" data) and from their own perspectives and intuition ("soft" data).

After deciding that their problem warranted the use of MCDA and V·I·S·A, the four graduates sat down and analysed their situation. They decided that the situation at hand required them to use a number of pre-emptive criteria to screen out alternatives. These were:

- That the apartment should not exceed \$250,000 in price.
- That the apartment had to be in the inner-city area and no further than 10 minutes walking distance from the furthest place of work.

After individually brainstorming for criteria, the four graduates arranged to meet together and decide on an initial set of criteria that would enable them to look for alternatives that met these initial criteria. The group considered the following factors were desirable in an apartment:

- Had a balcony to enjoy Wellington's beautiful weather and so the wide doors leading out to the balcony would allow fresh air to circulate more quickly through the apartment. This was important especially when the apartment went unused for long periods of time.
- Had one or two bedrooms.
- Was fully or semi-furnished as the apartment needed to be used straight away. Also provided ease of use.
- Had morning and/or afternoon sun both for intrinsic reasons (mood) and for the welcoming nature of the apartment. Afternoon sun was seen as more important as the graduates would be at work more often in the morning.
- Had amenities close by.
- Had a small number of tenants (preferably no greater than five) so that the apartment building remained quiet especially when they wanted to relax.
- Had parking as it adds value to the apartment and two of the four graduates drive. A car parking building that was within two minutes walk was felt to be another option to consider.
- Should have elevator access
- Be of solid construction and provide reasonable levels of security



What criteria should be used in the criteria tree? How should the criteria tree be structured? Use the focusing questions to determine a suitable criteria tree.

- Do criteria exist which can be used to evaluate the differences in alternatives? What are they? Think of all the factors that influence your decision, including subjective factors. Use the 4 methods described earlier, namely brainstorming, plus-minus-interesting points, pairwise comparisons, and repertory grid.

- Are some of these criteria pre-emptive? Can they be used to screen out alternatives? What are they?

- Will all the criteria help to assess the alternatives? Are some of the criteria non-differentiating? If so, should they be excluded?
  
- Does the situation or problem suit an Input/Output analysis, or Cost/Benefit analysis, or a more general tree structure analysis (with more than 2 first-level criteria)? For the appropriate tree structure, group your criteria under the appropriate first-level criteria.
  
- Do the selected criteria fall into a logical hierarchical sequence? Draw it. Keep in mind the goal or objective of the decision - and use this to keep your tree as small and focused as possible. For an Input/Output or Cost/Benefit model, use these two top headings as the two top-level criteria in your tree. (Refer back to the School IT example, or the House purchase example for examples of cost/benefit models.)

## Answer

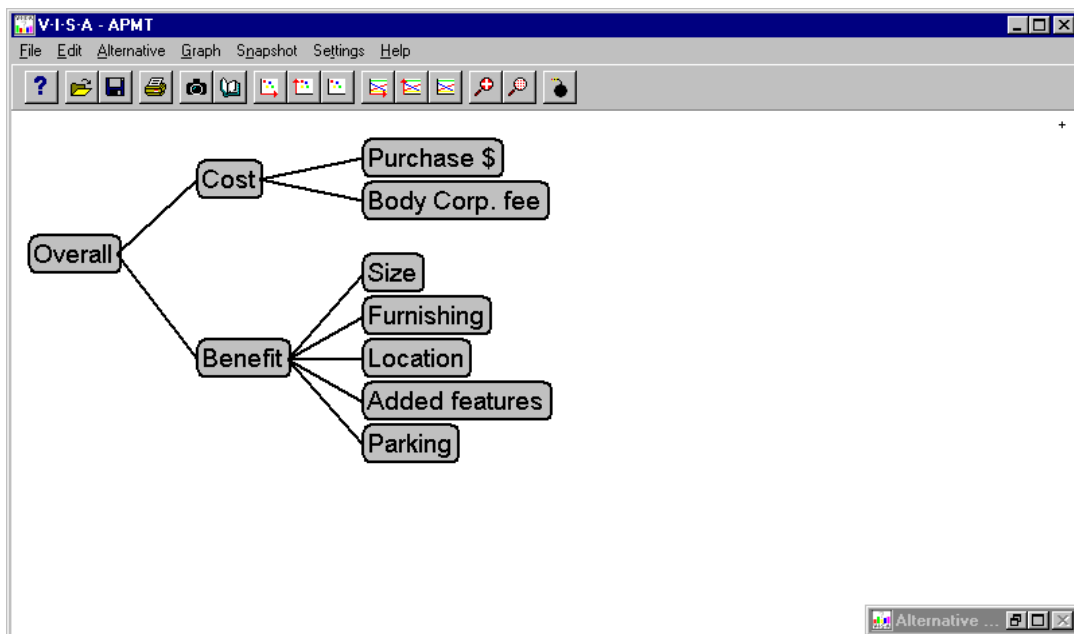
Through applying a screening process, two parent criteria and seven child criteria were selected for analysis. Ideally, the four graduates saw the problem as a cost/benefit analysis. Subsequently the parent criteria were Cost and Benefit. The sub-criteria, or child criteria, under the Cost criterion were selected as:

- The Purchase price of the apartment (*Purchase \$*)
- The Fee incurred through using an agency (*Body Corp. fee*).

The sub-criteria under the Benefit criterion were selected as:

- The *Size* of the apartment (square meters).
- *Furnishing* - how well the apartment is furnished: eg fully furnished, semi furnished or unfurnished.
- The *Location* of the apartment relative to each graduate's place of work.
- *Added features* - balcony, sun, number of tenants, amenities close by.
- *Parking* - Access and convenience of parking, ranging from on-site parking, car parking building nearby (2 minutes walk) to no parking.

Using the above criteria, develop the criteria tree and save the V·I·S·A model as 'Apmnt'. You will continue to develop this model through the rest of this module.



Having identified the pre-emptive criteria and the list of desired criteria, the group went out and researched and developed a set of apartment alternatives that met the pre-emptive criteria, and looked good against their desired criteria. In all ten apartments were considered:

**City 1** - One bedroom apartment situated on the fringe of the CBD. Your own private balcony is perfect for enjoying breakfast in the sun. A secure car park adds to the attraction of this unit. \$166,000. Body Corporate (BC) fee \$1,400.

**City 2** - Two-bedroom apartment located handy to the central city, afternoon sun. \$146,000. BC \$1,200.

**City 3** - This two-bedroom apartment has a balcony, which gets good morning sun, and is also semi-furnished. \$227,000. BC \$2,000.

**City 4** - Prime location! One bedroom apartment situated perfectly for the busy executive. Semi-furnished with all whitewear. \$182,000. BC \$2,000.

**City 5** - Avoid the traffic hassles. One bedroom apartment enjoying afternoon sun. \$136,000. BC \$1,000.

**City 6** - This one bedroom apartment is situated in a top location. Good morning sun. \$142,000. BC \$1,150.

**City 7** - Just move in! This two-bedroom apartment is situated handily to the CBD and is fully furnished. Secure car parking and a great balcony are just two of the additional features. \$250,000. BC \$2,000.

**City 8** - Well situated, this two-bedroom apartment has a car park, is semi-furnished, and enjoys great all day sunshine. \$225,000. BC \$1,800.

**City 9** - Two-bedroom apartment located on the fringe of the city. Secure car park and a balcony are two of its features. Semi-furnished is another bonus. \$250,000. BC \$1,750.

**City 10** - Enjoy the afternoon sun from your balcony when you buy this two-bedroom apartment. Good location. \$146,000. BC \$1,300.

After visiting each apartment the group were able to apply the other methods (plus-minus-interesting points technique, pairwise comparisons, and repertory grid) for eliciting criteria to check whether they'd missed anything important. They were also able to eliminate potential criteria that were inappropriate or non-differentiating. The non-differentiating criteria included elevator access, the type of construction used and security requirements. In eliminating these non-differentiating criteria,

the group were mindful that these criteria may require including in the model if any alternatives were added later which did not meet these requirements.

They were now ready to begin using the V·I·S·A program....

- K Using the 'Apmnt' model you developed earlier, open the Alternatives window and create 10 alternatives. Rename each alternative City 1, City 2, City 3, ..., City 9, and City 10. Save your model again.

	Purchase \$	Body Corp. fee	Size	Furnishing	Location	Added features	Parking
City 1	0	0	0	0	0	0	0
City 2	0	0	0	0	0	0	0
City 3	0	0	0	0	0	0	0
City 4	0	0	0	0	0	0	0
City 5	0	0	0	0	0	0	0
City 6	0	0	0	0	0	0	0
City 7	0	0	0	0	0	0	0
City 8	0	0	0	0	0	0	0
City 9	0	0	0	0	0	0	0
City 10	0	0	0	0	0	0	0

Devise a suitable scoring system for each of the criteria using the following focusing questions:

- What data is available, is it representative of the situation?
- Do some of the criteria require qualitative or subjective scoring? If so, how will this be done?

- Does the data fit in well with the chosen criteria? Why?
  
  
  
  
  
  
  
  
  
  
- Does the available data appear to suit local or global interval scales? Why?
  
  
  
  
  
  
  
  
  
  
- Does the situation require reversing a few scales? Why? Which criteria?
  
  
  
  
  
  
  
  
  
  
- Should non-linear scales be used in some cases? If so, what should they be?

- Can each of the criteria be scored on its own, without having to consider any of the other criteria? If not, the criteria may not be "preference independent" (see Goodwin and Wright), and the tree should be re-structured or the criteria redefined. Often it suggests that the interdependent criteria should be linked in some way, such as a parent-child relationship.

### ***Answer***

Through analysing the raw data, the graduates determined how each of the criteria would be scored and whether a global or local scoring system would be used. These were:

- Purchase \$ - the alternative price divided by the highest price then multiplied by 100. (Global-local scale)
- Body Corp fee - the alternative fee divided by the highest fee then multiplied by 100. (Global-local scale)
- Size - (alternative size - smallest size) / (largest size - smallest size). (Local scale)
- Furnishing - None 0, Semi 50, Full 100. (Local scale)
- Location - Fringe 0, Inner 50, Central 100. (Local scale)
- Added features - Balcony +35, Morning sun +15, Afternoon sun +25, Tenants (<5) +10, Amenities close by +15. (Global scale)
- Parking - none 0, public 50, on-site 100. (Local scale)

*The scales for Purchase price and Body Corporate fees are a mixture of global and local scales. The maximum is set by the maximum figure, but the lowest goes all the way to zero. If another apartment is chosen that is more expensive, they'll need to reset the scales, so it's probably more of a local scale than global. Furnishing etc is defined as "local" because they're based on the range of options being considered, rather than those available in the whole apartments market.*

## *Questions*

Do you think the above scoring systems are suitable and representative?

Do they differ from your own? Why? Suggest any changes you might make and why you would make these changes.

For now we will leave the scoring system devised by the graduates as is. Using the scoring systems provided above, calculate the scores and enter them into the appropriate cells in the spreadsheet on the next page. The raw data is also provided on this page. Then open a new Excel workbook and copy the scores onto the Excel spreadsheet (Remember to follow the instructions given on page 38 carefully: cell A1 must be blank, Row 1 should contain the criteria headings, and column A the alternatives' names, with data starting in cell B2.) Save the new spreadsheet as 'ApmntScores' and save as a .txt (text tab delimited) file. Close the Excel file.

Open your Apartment model within V·I·S·A and import the ApmntScores.txt data file through the *File/Import/EXCEL TXT file* pull down menu option. Check the scores appear in the Alternatives widow. Save the Apartment model.



**Original Data**

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1	\$166,000	\$1,400	175.0	No	Fringe	Bal,Mor,Ame	On-site
City 2	\$146,000	\$1,200	190.0	No	Inner	Aft,Ame	Public
City 3	\$227,000	\$2,000	187.5	Semi	Fringe	Bal,Ten	None
City 4	\$182,000	\$2,000	165.0	Semi	Center	Mor,Ten	Public
City 5	\$136,000	\$1,000	150.0	No	Fringe	Aft	None
City 6	\$142,000	\$1,150	157.5	No	Center	Mor,Ame	Public
City 7	\$250,000	\$2,000	195.0	Full	Inner	Bal,Ten,Ame	On-site
City 8	\$225,000	\$1,800	200.0	Semi	Inner	Mor,Aft	On-site
City 9	\$200,000	\$1,750	190.0	Semi	Fringe	Bal,Ten	On-site
City 10	\$146,000	\$1,300	187.5	No	Inner	Bal,Aft,Ten,Ame	Public

**Scores**

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1							
City 2							
City 3							
City 4							
City 5							
City 6							
City 7							
City 8							
City 9							
City 10							

Having inputted the scores, the next step is to determine a set of appropriate weights. Use an appropriate method to determine the weights for each of the sub-criteria in your Apartment model.

**Answer**

Table of scores:

	Purchase \$	Body Cop. fee	Size	Furnishing	Location	Added feat.	Parking
City 1	66	70	50	0	0	65	100
City 2	58	60	80	0	50	40	50
City 3	91	100	75	50	0	45	0
City 4	73	100	30	50	100	25	50
City 5	54	50	0	0	0	25	0
City 6	57	58	15	0	100	30	50
City 7	100	100	90	100	50	60	100
City 8	90	90	100	50	50	40	100
City 9	80	88	80	50	0	45	100
City 10	58	65	75	0	50	85	50

The four graduates, in deciding which weights to assign each of the criteria in their V·I·S·A model, began with the criteria under the Cost criterion. They noted that while the Purchase price of an apartment is a one-off cost, the associated Body Corporate fee is an annual fee. These costs need to be put on the same footing, eg annual costs, and one way of doing that is to replace Purchase prices by annual mortgage costs (strictly one should add insurance costs etc, but let us assume that they will be the same for all apartments, ie a non-discriminating factor). Next, they examined the ranges represented by the scoring scales for Purchase price and Body Corporate fee. The Purchase price scale represents a range of \$250,000 in purchase prices. At current interest rates, this would equate roughly to a range of \$20,000 pa in mortgage costs. The BC fee scale represents a range \$2,000 pa. Hence a change of 5 points on the Purchase price scale is roughly equivalent to a change of 50 points on the BC scale. This provides a starting point for the relative weights. If there are other non-financial factors influencing the weights, they could then be incorporated, eg they may feel happier parting with \$1 towards a mortgage than \$1 towards the BC fee, in which case the weighting may be a bit less than 10:1. In the absence of any such indications, let's just use the relative values calculated above as the weights.

	Relative importance	Weight
Purchase \$ Annual Cost	100	0.91
Body Corp. fee	10	0.09
<i>Sum</i>	110	1.00

Next, the four graduates determined the weighting for each of the criteria under the Benefits criterion. The graduates used the swing weights method. First they imagined an alternative that scored the worst on all the Benefits criteria. It was the smallest, was unfurnished, was on the Fringe, had the fewest added features, and had no parking. If they were allowed to improve one of these criteria to its best value, which would it be? They chose parking, firstly because of the intrinsic worth of having convenient parking, as they would be using the Apartment when visiting Wellington from outside the city, and secondly, the difference between the parking facilities offered by the apartments was considered very significant. Furnishing was deemed to be the next most important criterion, followed by Location (although Location is usually intrinsically important, the graduates felt there was little real difference between the Apartments on offer in terms of Location). The Size and Added Features of the Apartment were judged to be of last equal importance. Using the swing weights method, the graduates agreed upon the following set of weights:

	Relative Importance	Weight
Parking	100	0.38
Furnishing	80	0.31
Location	40	0.15
Size	20	0.08
Added Features	20	0.08
<i>Sum</i>	260	1.00

*So summarizing the steps in deciding Weightings:*

1. *First decide on criteria scoring scales.*
2. *For each criterion, identify what range of values is represented by the 100-point VISA scale.*
3. *Rank the criteria in importance, based on the value to you of a swing from worst to best on each of these ranges.*
4. *Assigning a value of 100 points to the most important criterion, work out the relative importance of the other criteria.*
5. *Calculate the normalised weights.*

*Questions:*

- Using the Apartment model, which alternative has the best Overall score?
- Does it make sense to use the Overall score? Why not?
- Which alternative is best in a Cost/Benefit plot? Why?
- Which alternative is the better option to take?
- In the Cost/Benefit plot, which alternatives are being dominated?
- Which alternatives lie on the "efficient frontier"?

## ***Answer***

In this case, it doesn't make sense to use the Overall scores, as all this will favour the apartments with the highest benefits and the highest costs. The former is desirable, but the highest cost is undesirable. So for this sort of model where Cost has been scored this way around, the Overall scores should be disregarded and the X-Y plot of Cost-Benefit will give the best measure.

Specifically:

- City 7 has the highest Overall score. But this is meaningless, as although City 7 has the highest Benefits, it also costs the most.
- Hence the Cost-Benefit plot will be more useful, given that we've set the scores for cost with a high cost getting a high VISA score, and a low cost getting a low VISA score.
- From the Cost-Benefit Plot, and with their original set of weights, City 7 actually has the highest Benefit-Cost ratio.
- The alternatives on the efficient frontier at this level are City 5, 6, and 7, but few people would be expected to choose City 5 over City 6, since City 6 has far greater benefit for very small additional cost.
- City 3 is clearly dominated by several apartments that score better in terms of both Benefits and Costs.
- City 2, 6 and 10 almost coincide on the Cost-Benefit plot. They all provide similar value for money. City 2 and 10 are just dominated by City 6, but are so close to it, they are effectively the same.
- City 1, 4, 9 and 8 lie just below the Efficient Frontier. Belton (1990, p74) warns us to beware of discarding them prematurely: while they appear to not be on the Efficiency Frontier, they may still be good all-round alternatives. They may fall into what Zeleny calls the Linearity Trap (see Belton, p74).
- Apart from City 3 and City 5, most of the apartments give similar benefit for each dollar spent - and the deciding factor may well be how much the graduates want to pay, or some other criterion outside the model!

*{These answers may change depending on the set of weights used for the model}.*

*NB If you find it easier, you can reverse the scoring scales at the outset, as is shown in the following 2 charts. Then you CAN use the Overall scores, as well as X-Y plots..*

**Scaling: Purchase \$** [X]

Select the type of scaling you want to use for scores for this criterion.

Linear Scale  
 Non-Linear Scale

Best:   
Worst:

Units:

Qualitative (Discrete)  
Select the required scale in the box below:

**Scaling: Body Corp. fee** [X]

Select the type of scaling you want to use for scores for this criterion.

Linear Scale  
 Non-Linear Scale

Best:   
Worst:

Units:

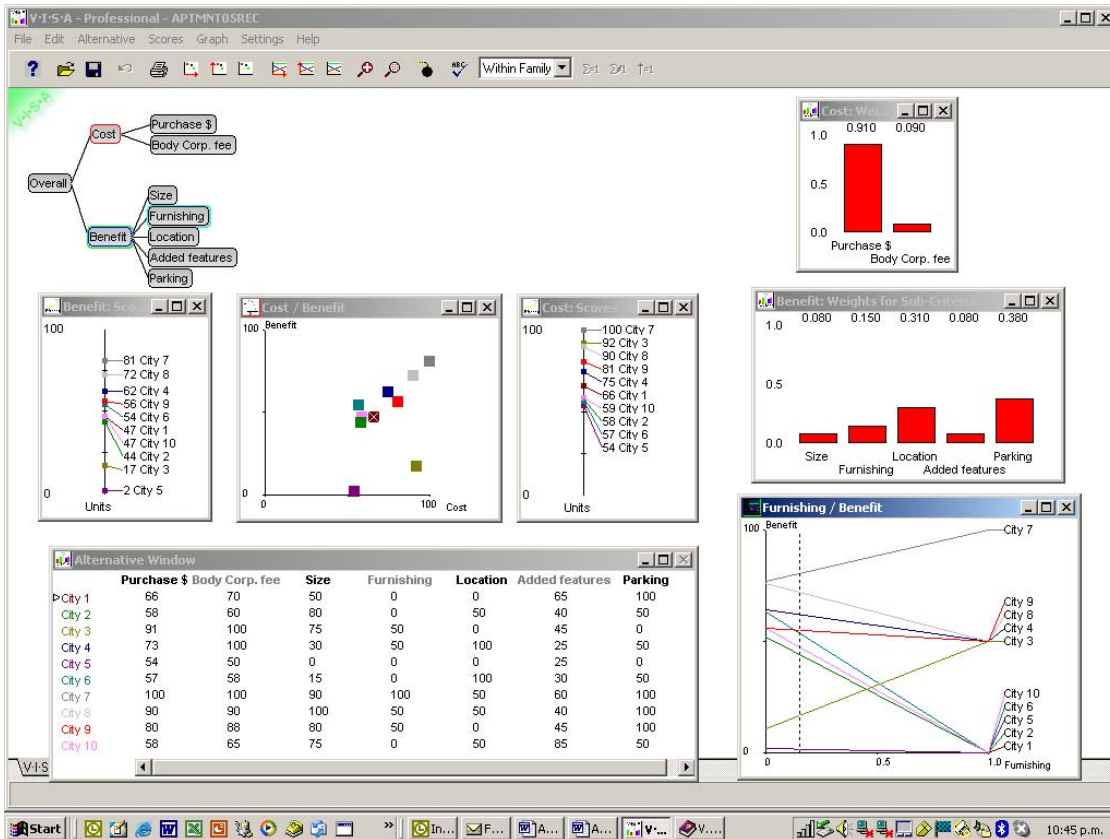
Qualitative (Discrete)  
Select the required scale in the box below:

Having determined through an X-Y plot that Apartments 2, 6 and 10 seem the best alternatives to select, given that they would rather opt for the lower priced apartments if reasonable, the graduates remained uncertain as to how accurate their weighting system was, and how robust these options would be to changes in the weights. To check the impacts of this uncertainty, the graduates decided to perform a Sensitivity Analysis.

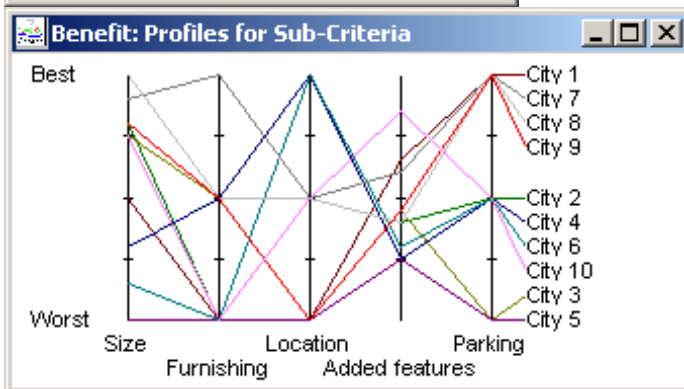
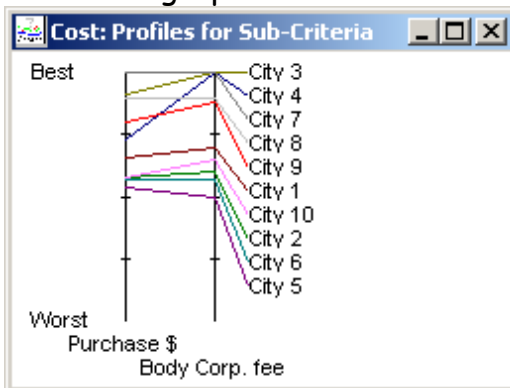
The graduates decided to plot the profiles charts for Costs and Benefits, and Sensitivity graphs of Purchase Price weight Vs. Cost score and Furnishing weight Vs. Benefit score.

This showed that if the weight given to Furnishing was reduced, then City 6 would perform better on Benefits, and become the most attractive option in terms of relative efficiency (best Cost-Benefit ratio). The Efficient Frontier for the set of weights shown in the screenshot below comprises City 5, City 6 and City 7, as before. City 6 would again most likely be preferred to City 5 because for a very small increase in Cost, the Benefits increase dramatically. If the graduates can afford City 7, it does have higher benefits, but the graduates need to ask themselves whether the additional benefits gained by choosing City 7 over City 6 justify the additional cost. Given this new set of weights, the graduate's preference for City 6 over City 7 now seems reasonable in that City 6 has a higher Benefit-Cost ratio.





- Profile graphs of the Cost and Benefit criteria.



Finally, they performed the DEA feature on each of the dominated alternatives.

Complete the above graphs and perform the DEA analysis.

Apply the following focusing questions and determine if the graduates' weighting system is robust.

***Questions:***

- Does the model behave as you'd expect it to as you change the criteria weightings?
- Are there some weights you are particularly unsure about? Where does this uncertainty lie?
- Are the linear and/or non-linear scales used suitable? Could they be altered or changed?
- Does a visual check of the weights reveal any changes in the order of the overall scores for each of the alternatives? If so, what are they?
- From the sensitivity graphs, over what ranges of weights does the best overall alternative remain the best? At what weights does Apartment 10 become 2<sup>nd</sup> best? In what circumstances would it not be best?

- Overall, do the results seem reasonable intuitively? If not, why not? Do you need to add a criterion or two? Or change the tree structure? Or revisit the scoring scales?
- If it does seem reasonable, does it enhance your understanding of your choices and preferences?
- Are the results clearcut or close? Is the preferred option a clear winner, or is there a close second? How close is the next best option?
- Are there apparent groups of options? Are there any general characteristics in common in these groups?
- Do the profiles and weighted profiles suggest any alternatives are dominated by other alternatives? What are they?
- If relevant, does the DEA feature suggest that a dominated alternative can be moved to the efficient frontier? If so, are the weight settings reasonable, or too extreme? Should further analysis be carried out to determine the significance of this finding?

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- Visual Thinking International Limited 1995: *V•I•S•A for Windows - User Guide*, Glasgow, United Kingdom
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<http://www.simul8.com/products/visa.htm>  
includes details on downloading a demo version of V•I•S•A, and pricing information
- Additional Information was used from the following Internet address:  
<http://www-bd.cricket.org/>

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## References

- <sup>1</sup> Belton, V 1990 page 53
- <sup>2</sup> Belton, V 1990 page 56
- <sup>3</sup> Belton, V 1990 page 55
- <sup>4</sup> Belton, V 1990 pages 61 - 62
- <sup>5</sup> Belton, V 1990 page 62
- <sup>6</sup> Belton, V 1990 page 62
- <sup>7</sup> Belton, V 1990 page 62
- <sup>8</sup> Belton, V 1990 page 69;  
Goodwin and Wright, pages 37-39
- <sup>9</sup> Goodwin and Wright page 21
- <sup>10</sup> Goodwin and Wright page 24
- <sup>11</sup> Belton, V 1990 page 69
- <sup>12</sup> Goodwin and Wright, 1998 pages 22-28
- <sup>13</sup> Goodwin and Wright pages 29-30
- <sup>14</sup> Belton, V 1990 page 71
- <sup>15</sup> Edwards and Barron, quoted in  
Goodwin and Wright, page 42
- <sup>16</sup> Goodwin, P and Wright, G 1991  
Daellenbach, H page 33;  
pages 506-507
- <sup>17</sup> Visual Thinking 1995 page 38
- <sup>18</sup> Visual Thinking 1995 page 37
- <sup>19</sup> Belton, V and Vickers, S. 1993 page 883